

# Universal Scaling Between Relaxation And Caged Dynamics In Glass-Forming Liquids, Polymers And Mixtures

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# Outline

- Background
- Numerical Simulations
- Comparison with experiments
- Conclusions

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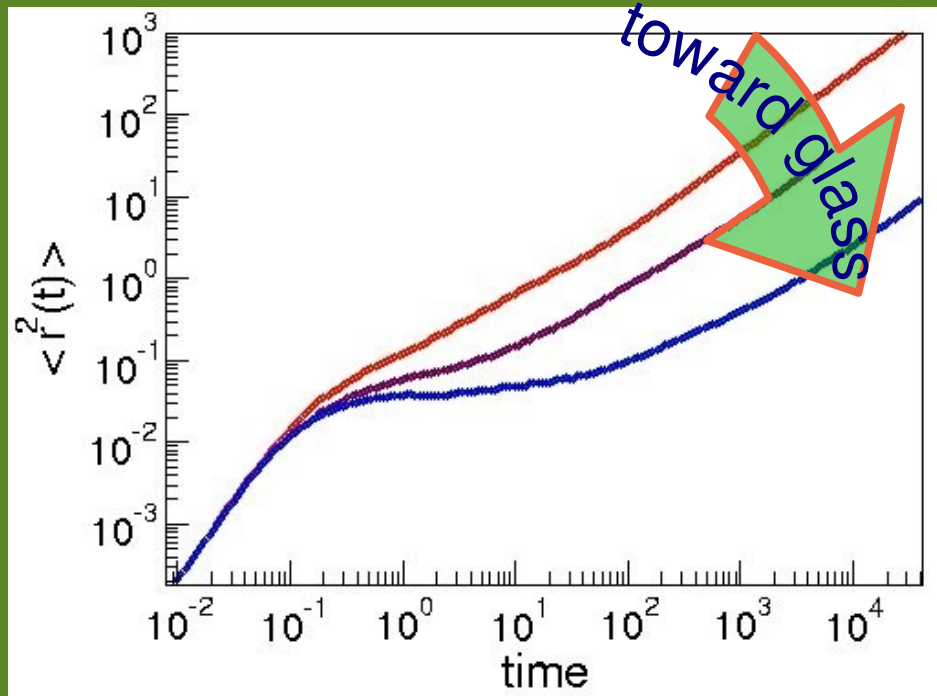
$$\approx 10^2 \text{ s @ } T_g$$

...but it is still intriguing

[Nemilov(1978), Brawer(1984),  
Dyre(1996), Scopigno(2003),.....]

# Focusing on the cage effects

MSD exhibits different regimes:  
ballistic    caging    relaxing

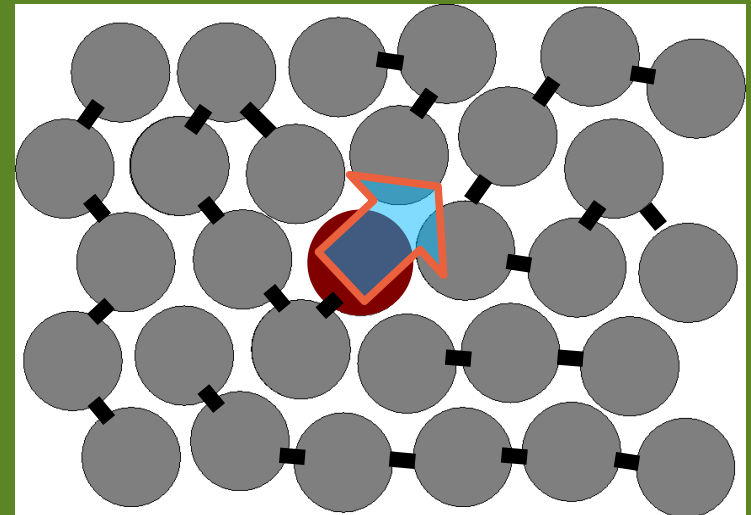
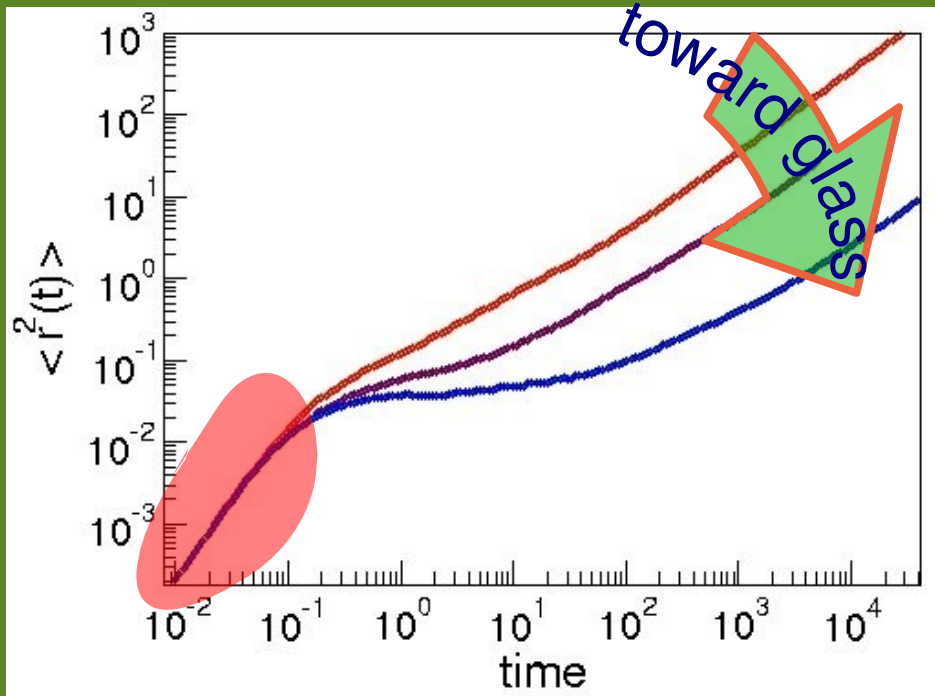




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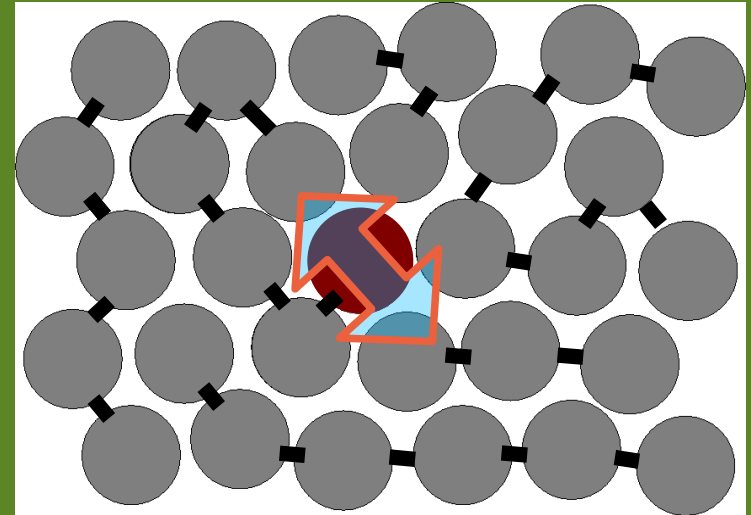
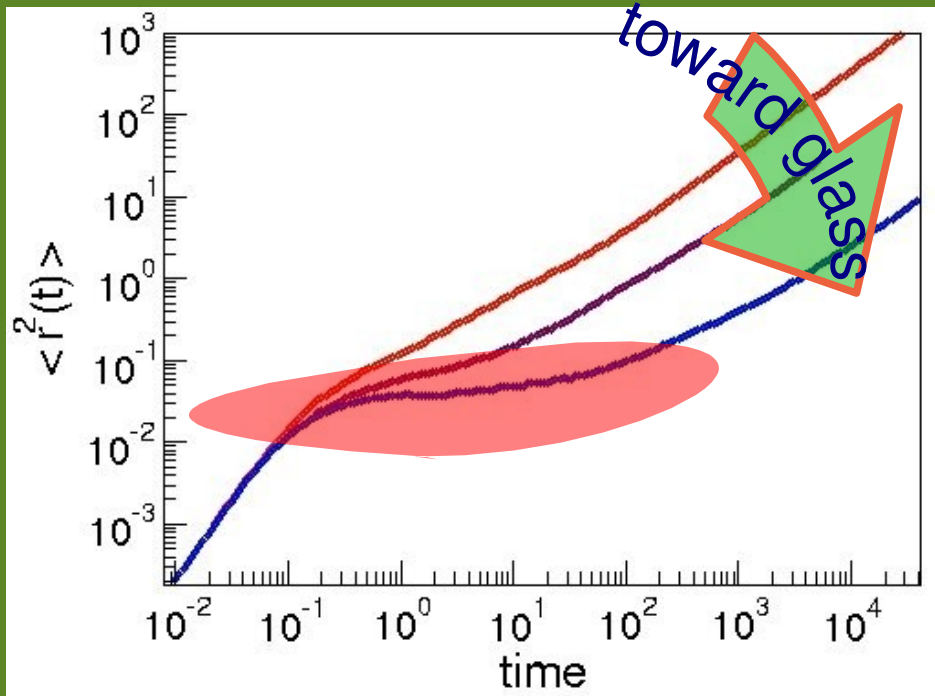
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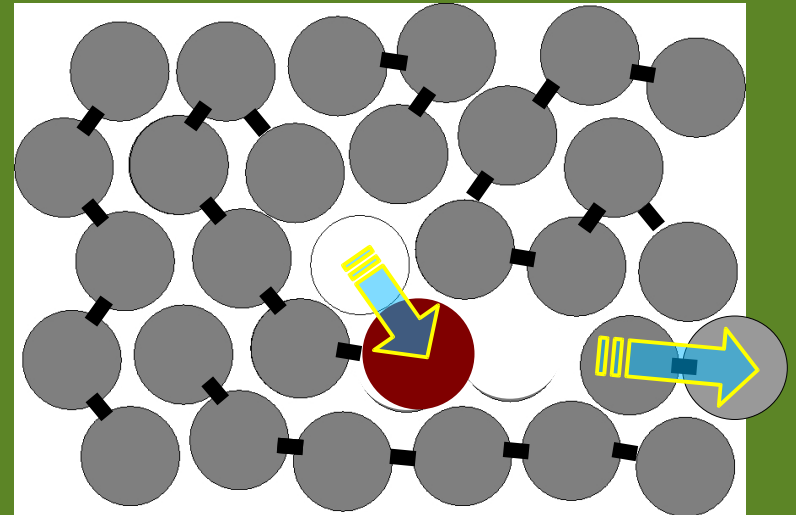
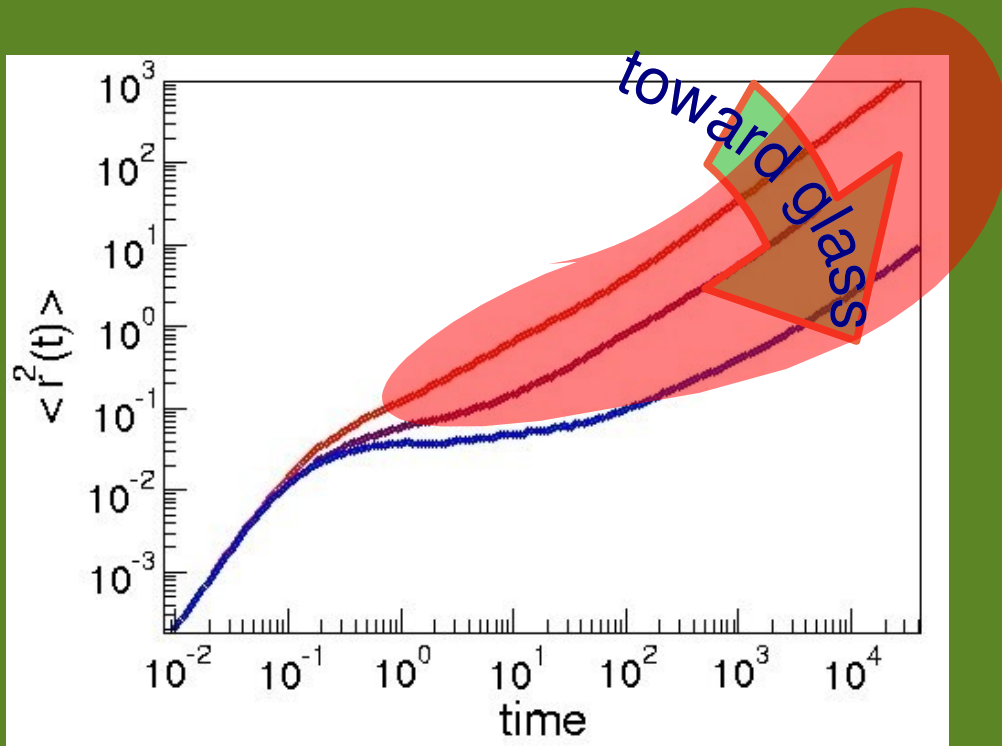
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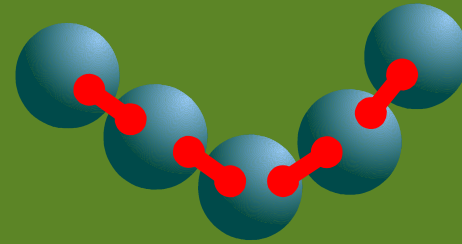


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# Models:

a) Bead-spring flexible polymer melt

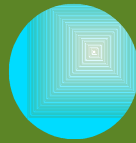
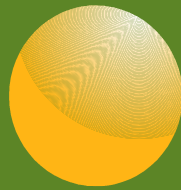
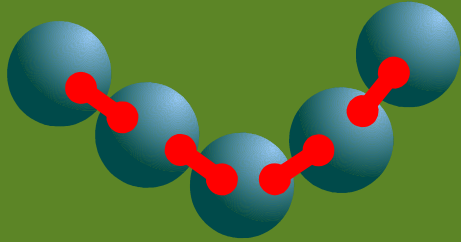


b) Modified Kob-Andersen Binary Mixture

- ◆ **80 : 20**
- ◆  $\sigma_B / \sigma_A = 0.88$
- ◆  $\sigma_{AB} / \sigma_A = 0.8$
- ◆  $m_B / m_A = 1$

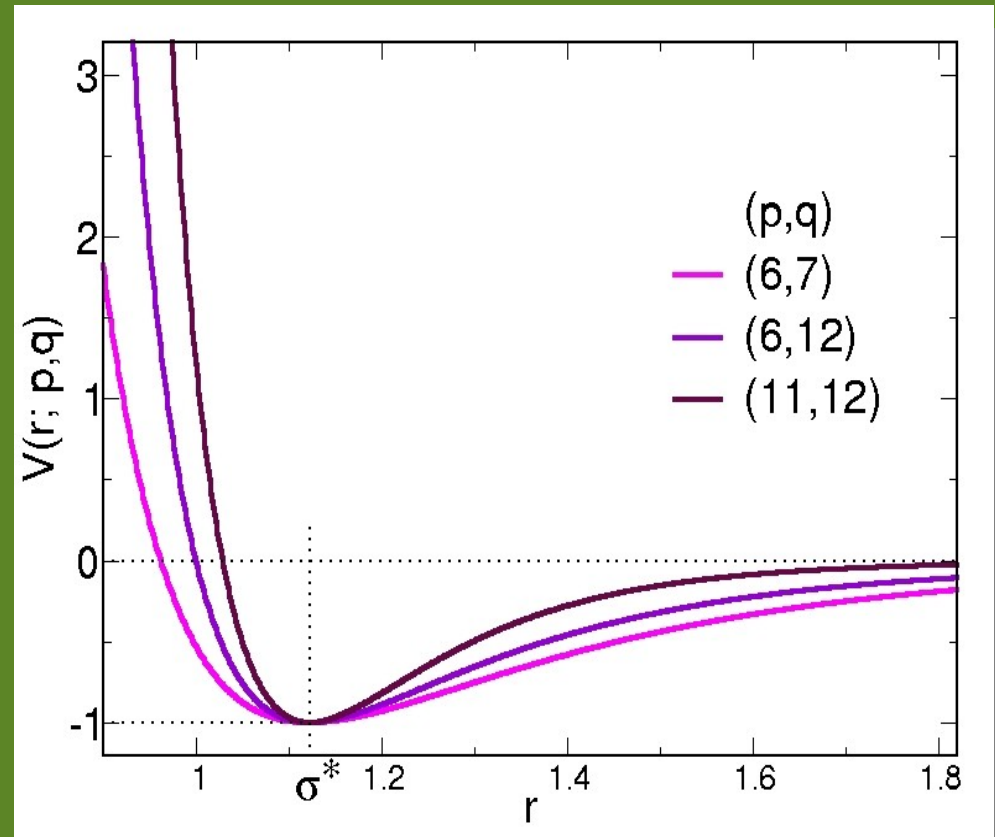


# Models : control parameters



$$V_{LJ}(r; p, q) = \frac{\epsilon}{q - p} \left( p \left( \frac{\sigma}{r} \right)^q - q \left( \frac{\sigma}{r} \right)^p \right).$$

- Density  $\rho$
- Temperature  $T$
- Chain length  $M$  (polymer)
- Potential softness:  $q$  and  $p$  exponents



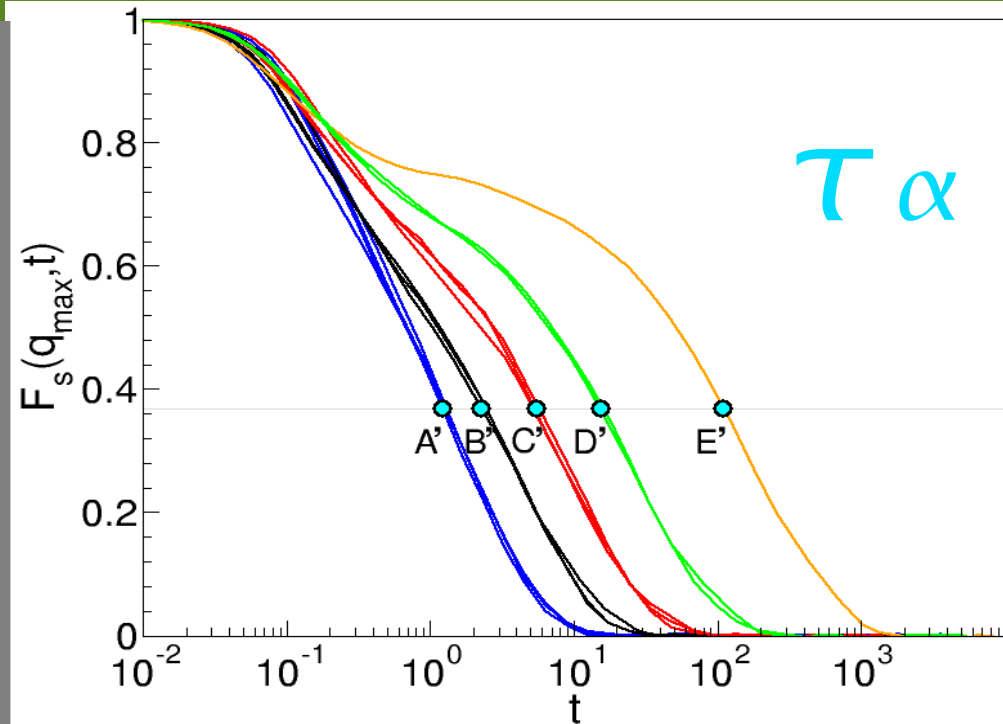
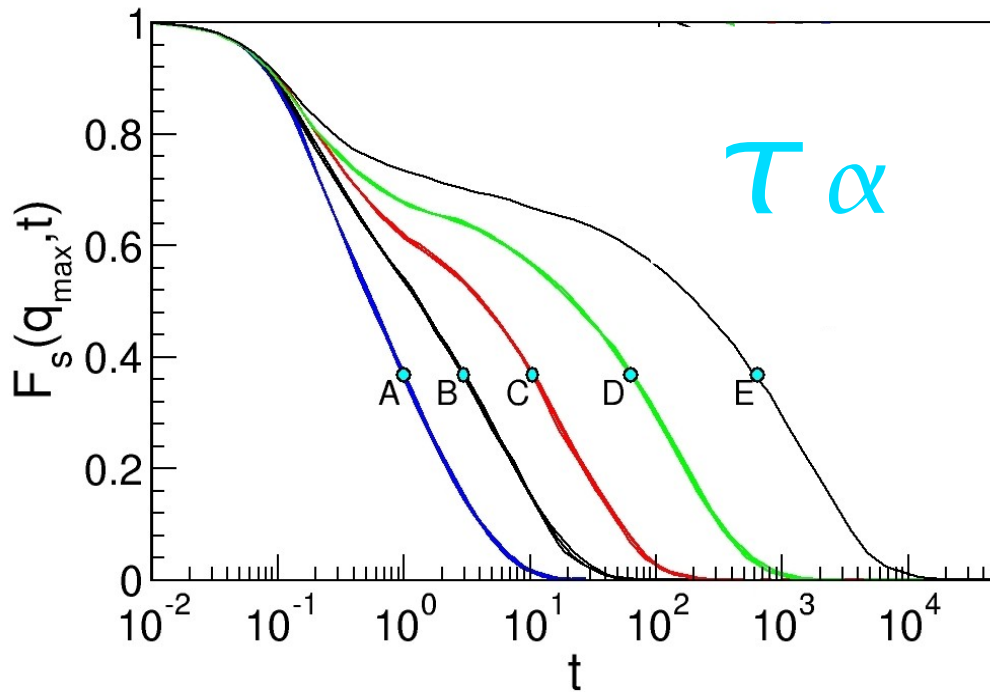
# Equivalent ISF States

DIFFERENT STATES RELAX IN THE SAME WAY!!!

They define EDM (Equivalent Dynamics Multiplets)

Polymers 

Binary Mixture 

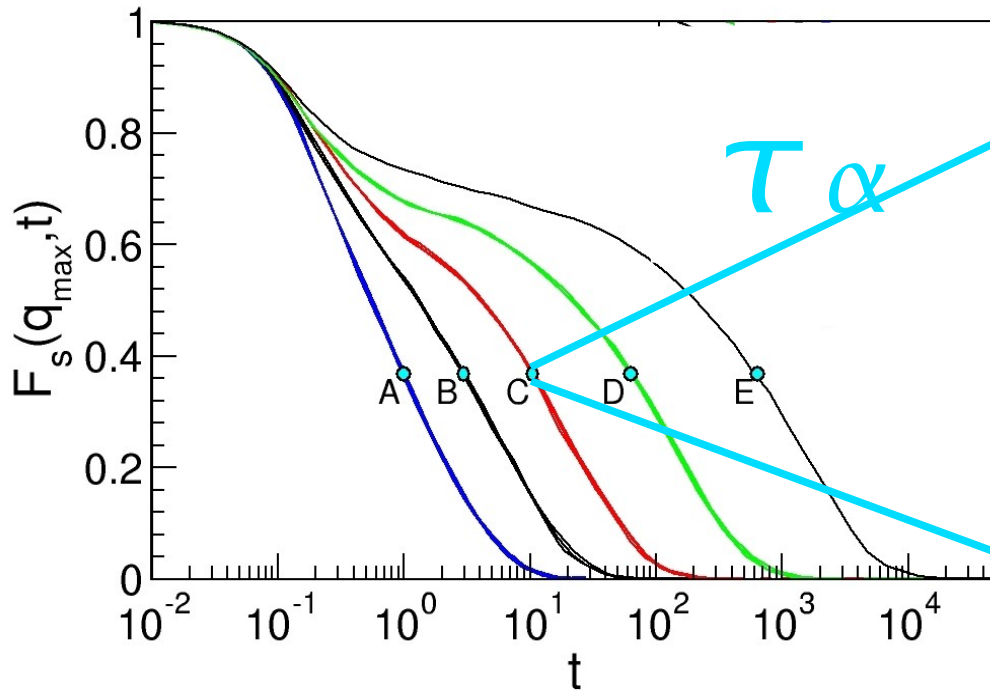


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Polymers 



{ M,  $\rho$ , T, q, p }

{ 2, 1.033, 0.5, 10, 6 }

{ 3, 1.056, 0.7, 12, 6 }

{ 5, 1.033, 0.6, 12, 6 }

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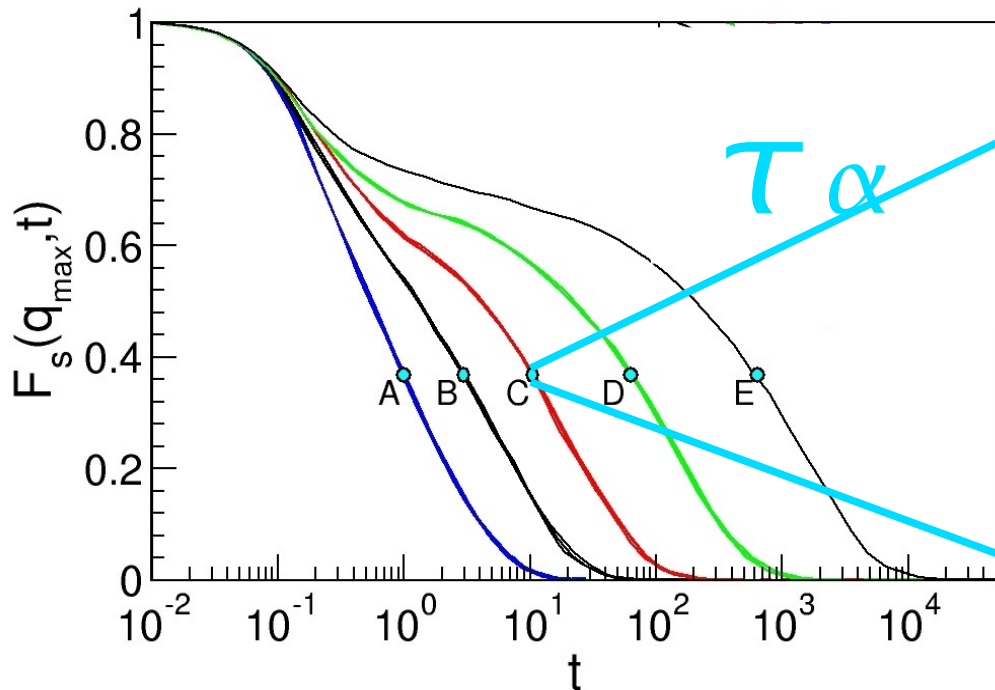
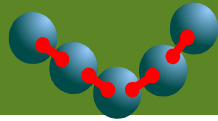


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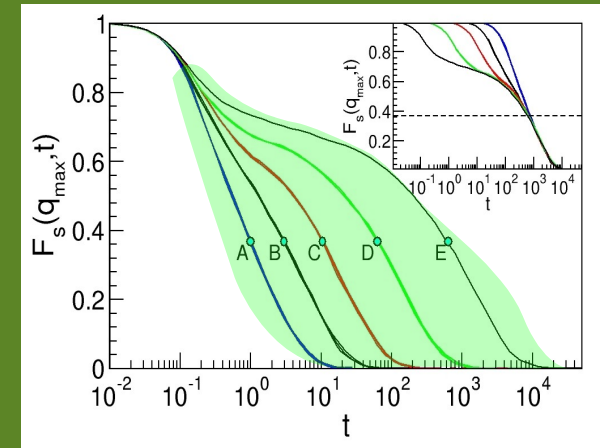
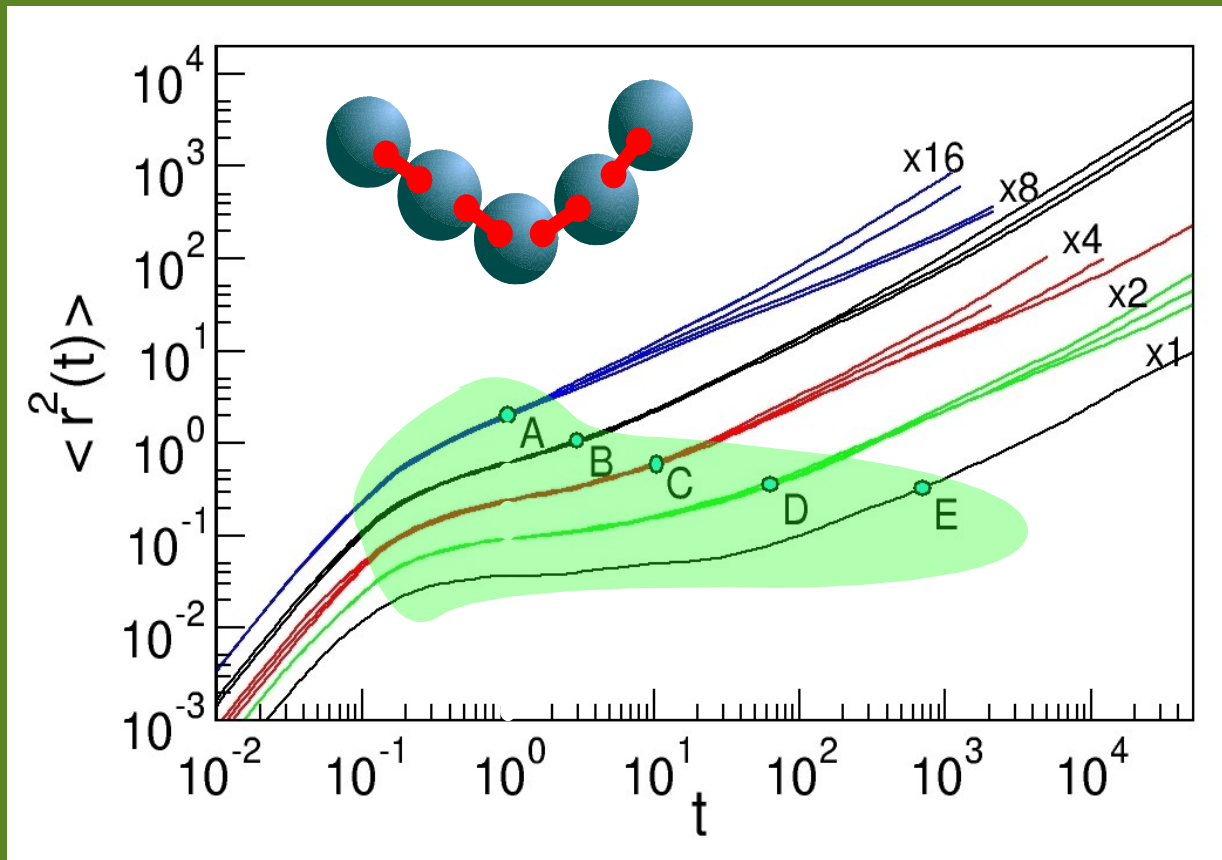
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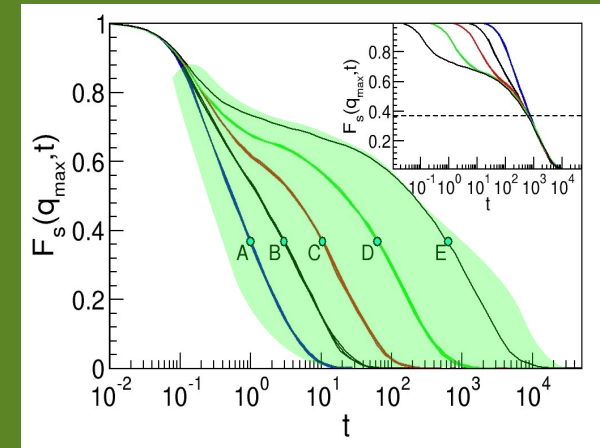
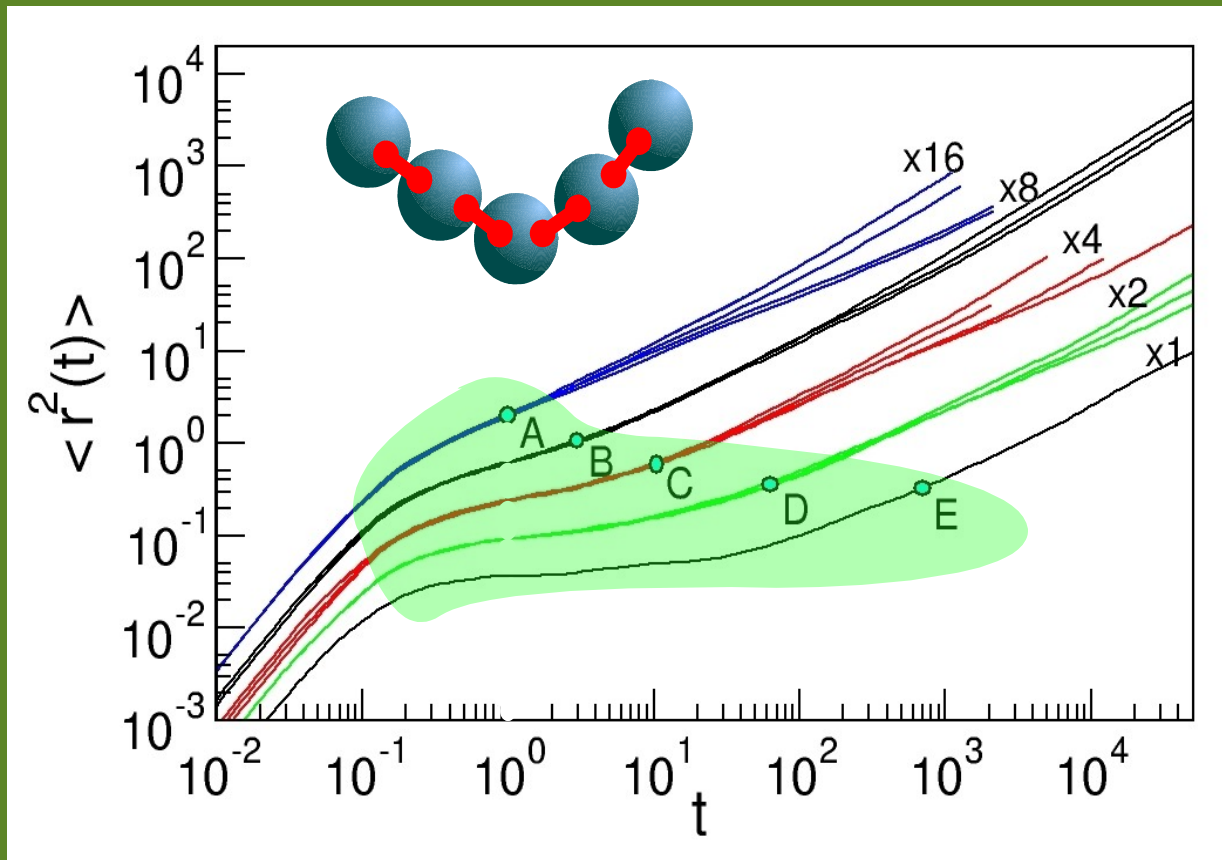
Then  $\tau_\alpha$  tags the  $\alpha$ -relaxation  
of EDM (Equivalent Dynamics Multiplets)

# Monomer displacement for EDMs



States with coincident  $\alpha$ -relaxation time exhibit coincidence in MSD from the **end of ballistic** until the **relaxation time** (in Polymeric Liquid)

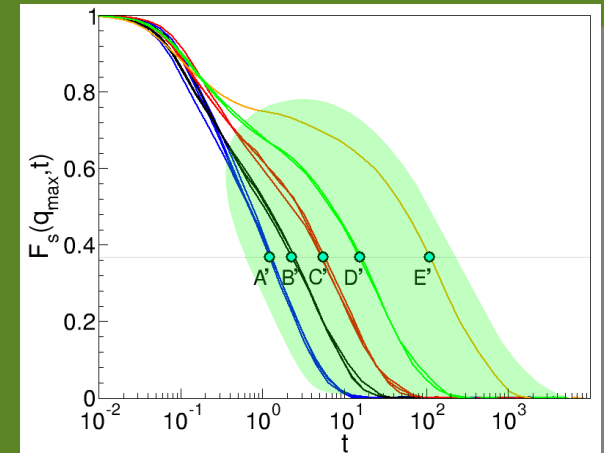
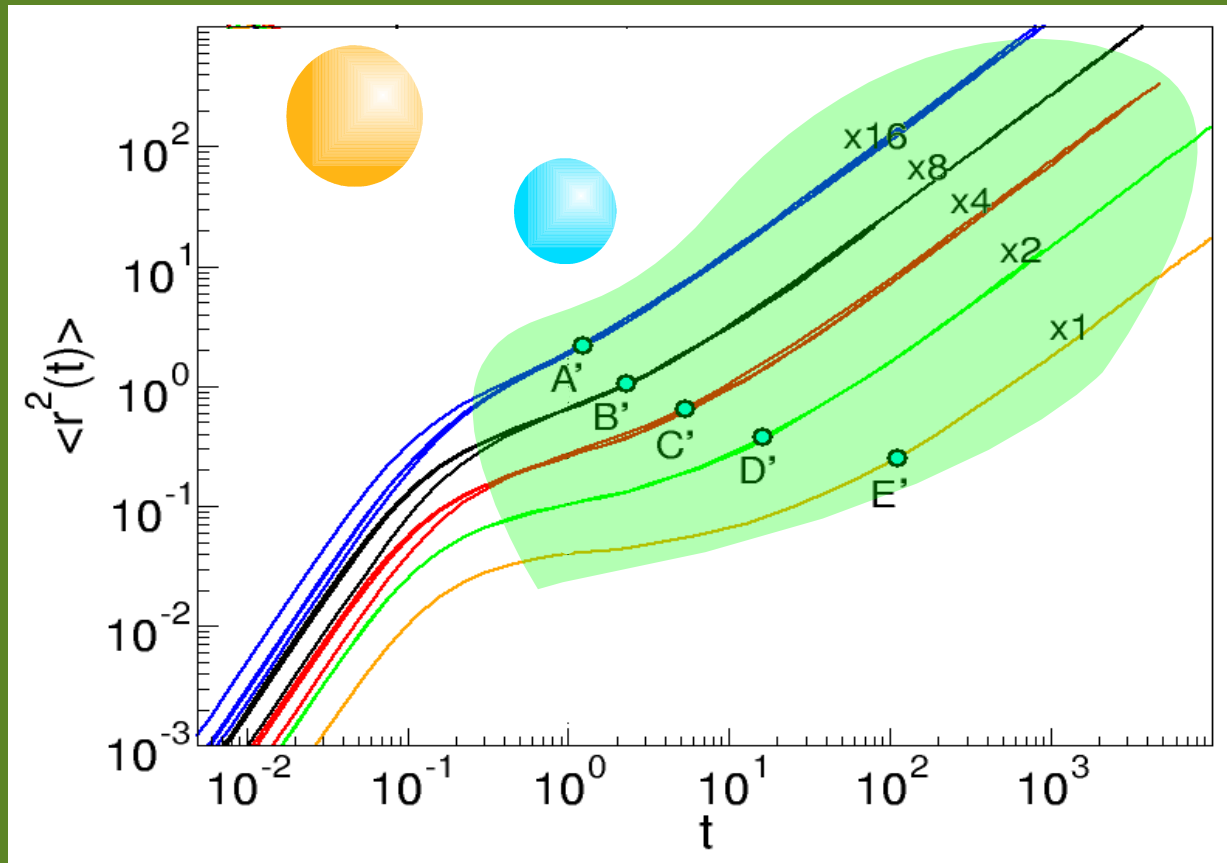
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VIBRATIONAL AND RELAXATION DYNAMICS ARE STRONGLY CORRELATED !!!

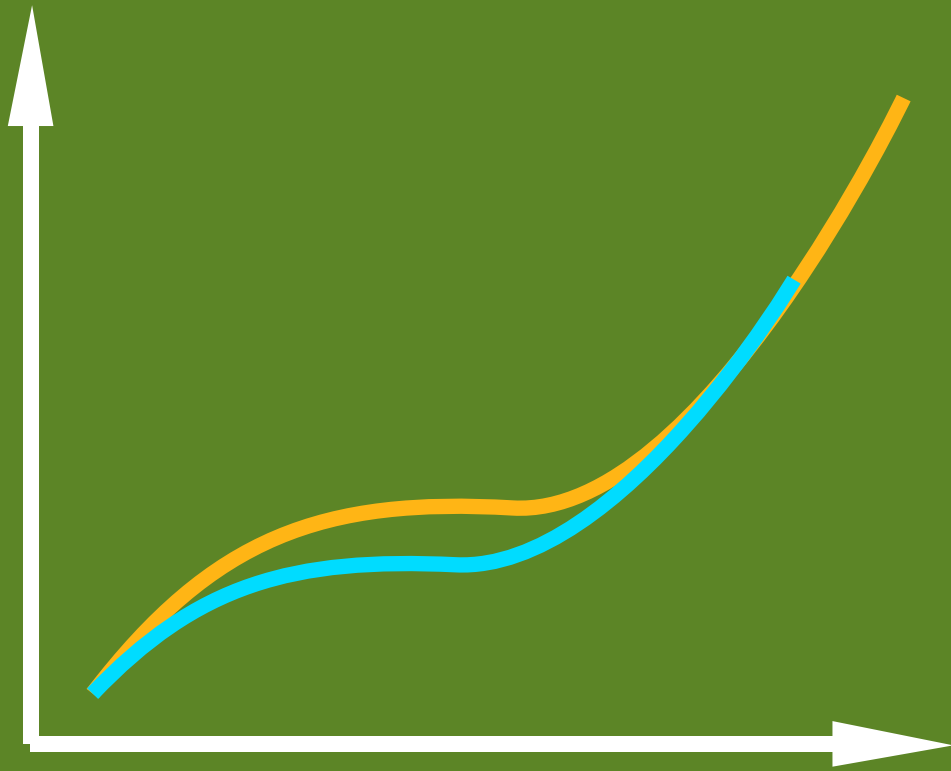
# ... more over for Binary Mixtures



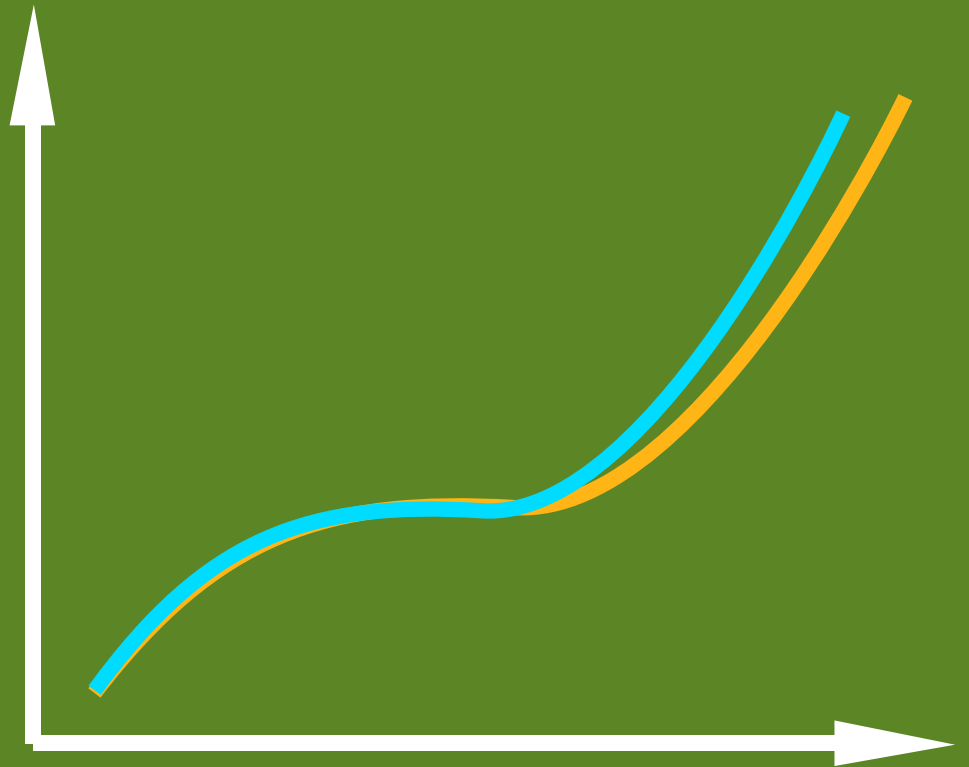
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VIBRATIONAL AND DIFFUSIONAL AND RELAXATION DYNAMICS ARE STRONGLY CORRELATED !!!

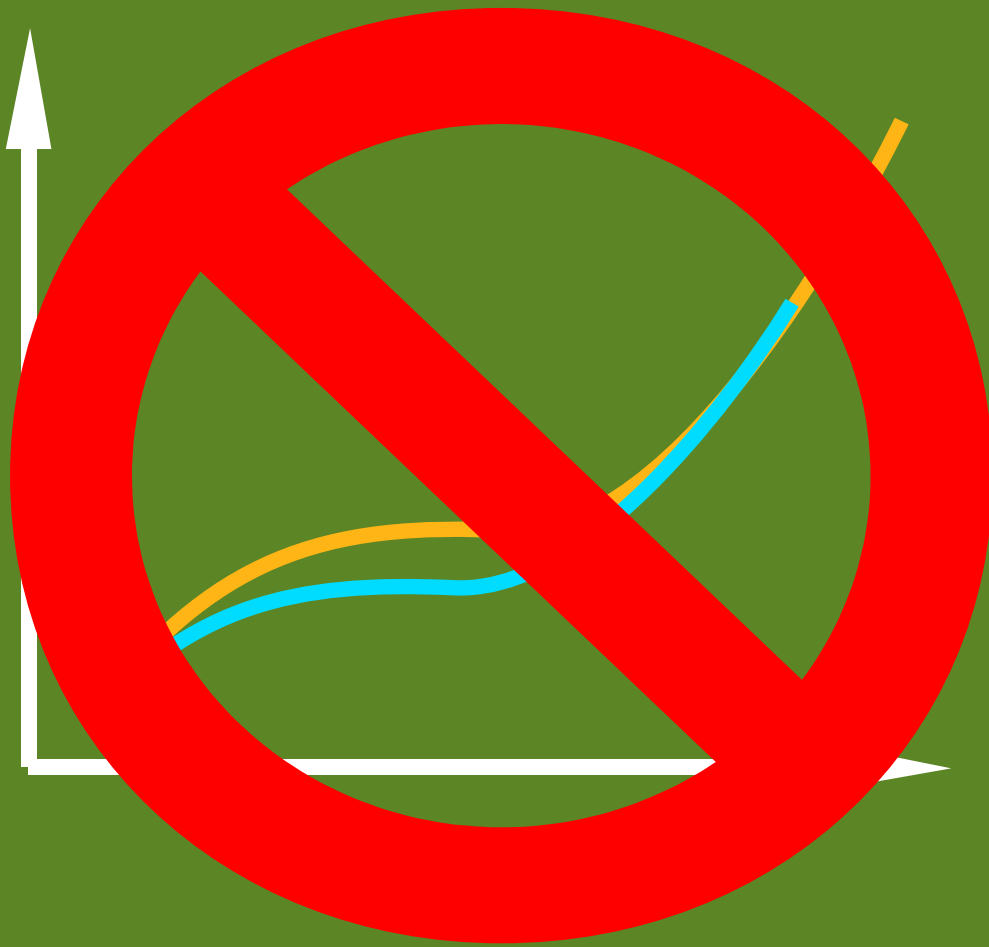
# Why do we care about EDMs ?



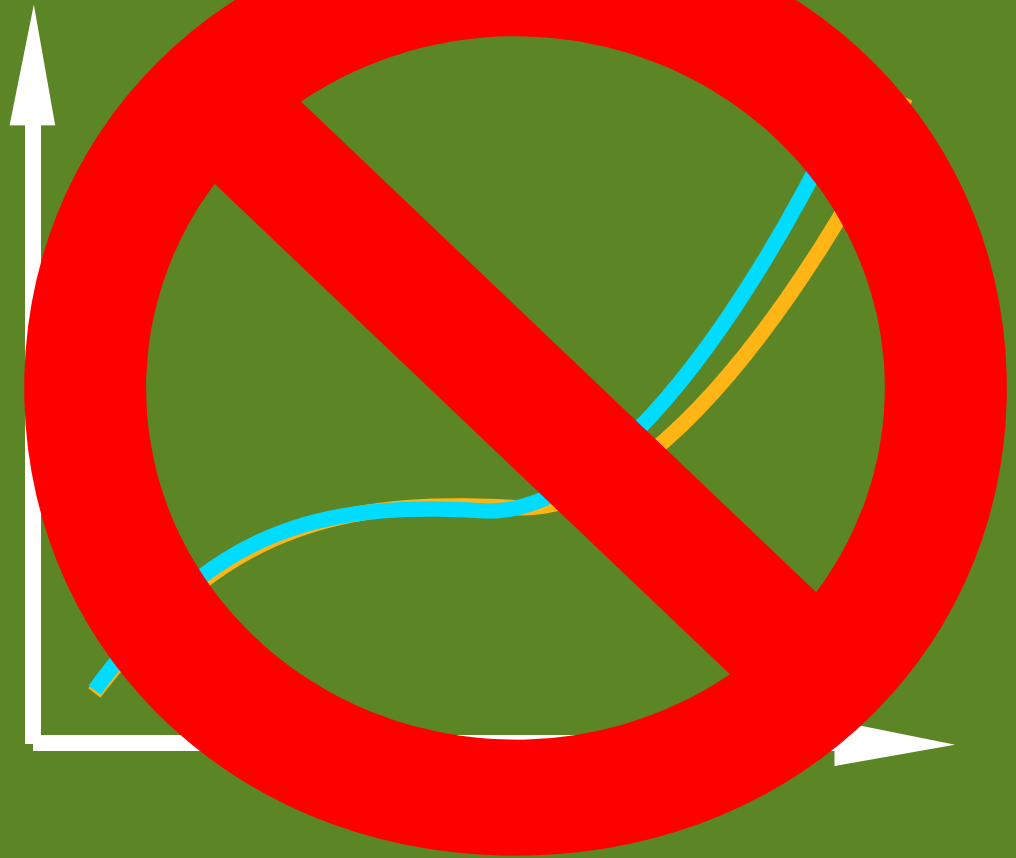
MSD



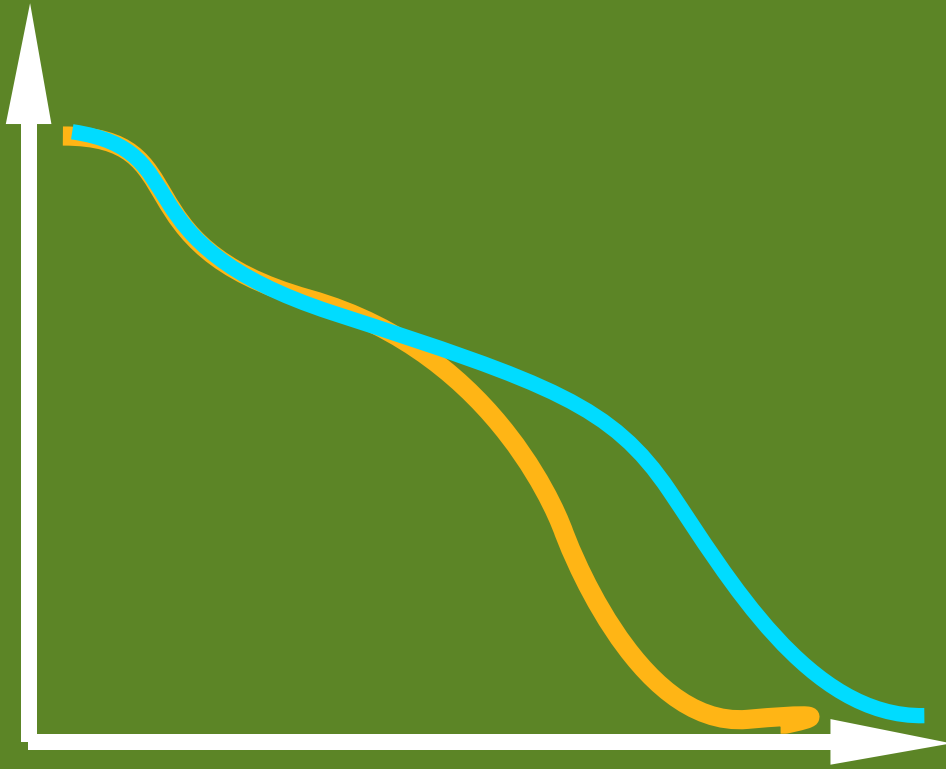
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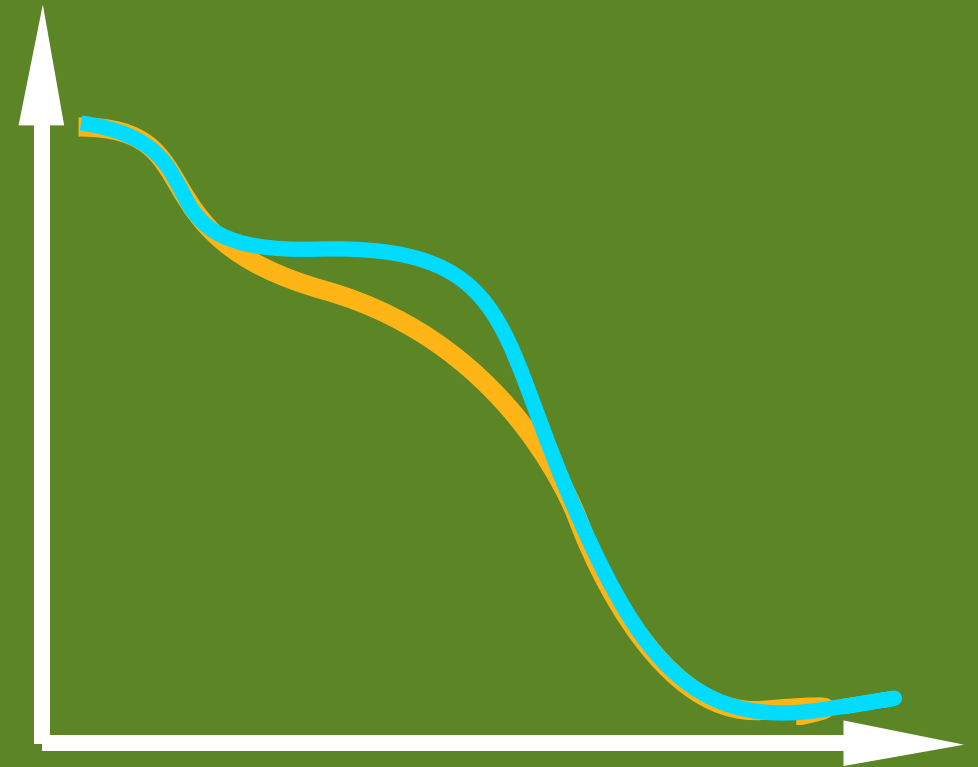
MSD



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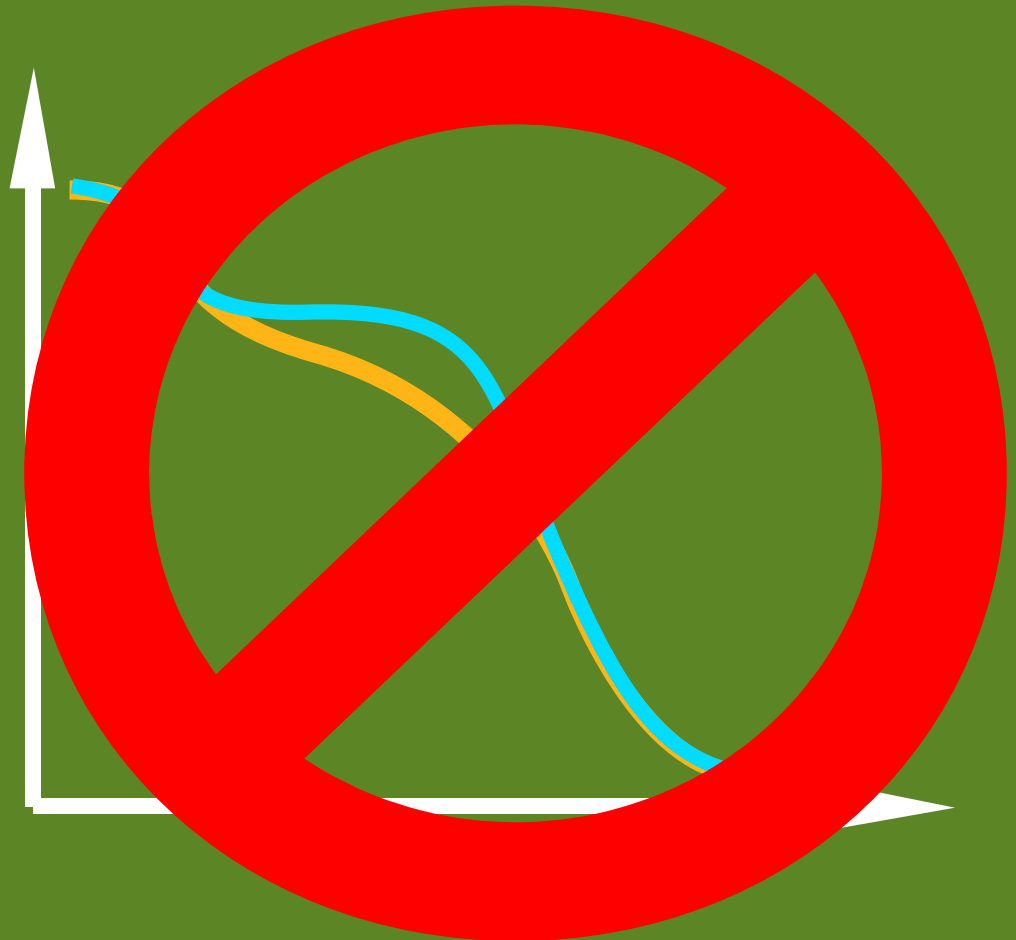
ISF



Why do we care about EDMs ?



ISF

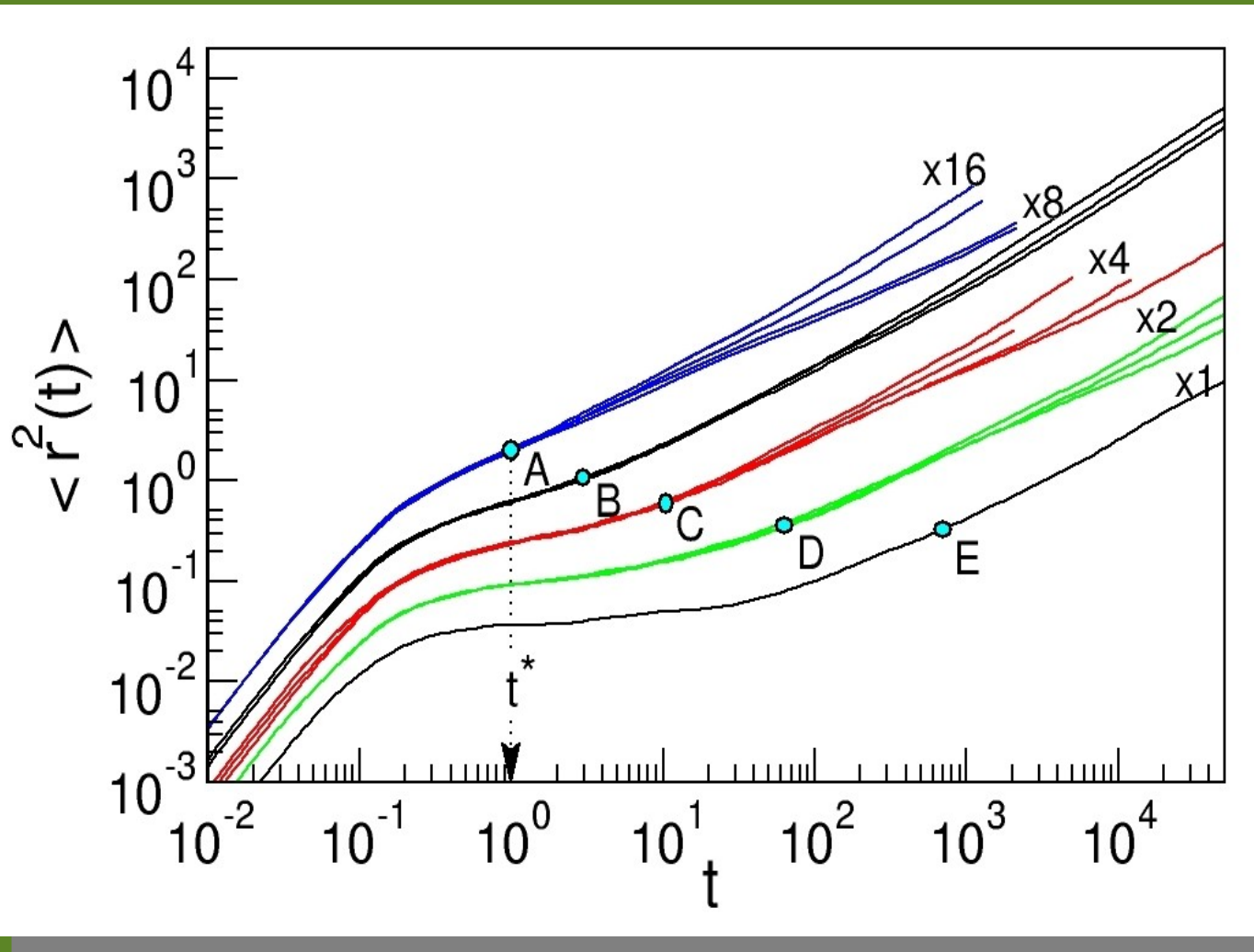




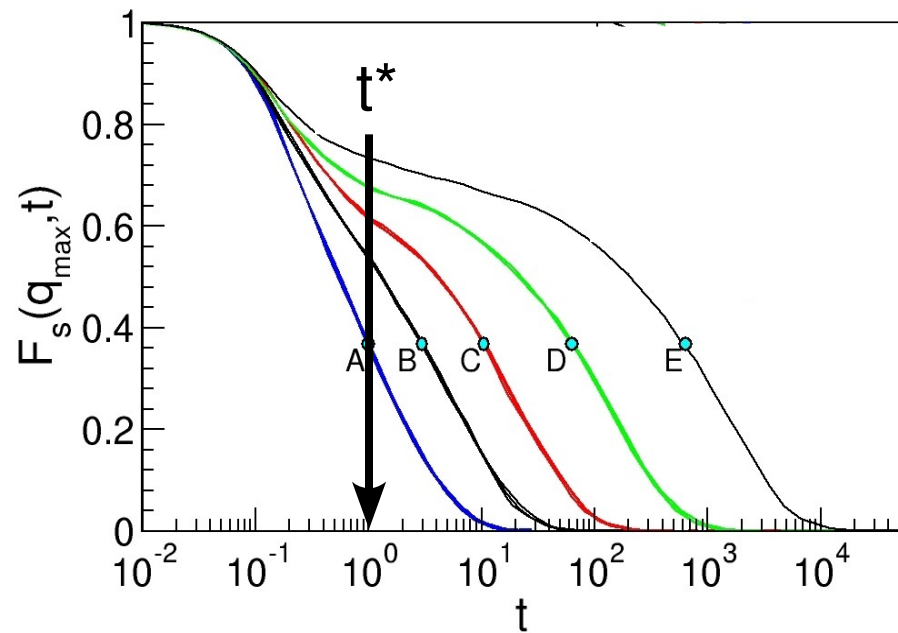
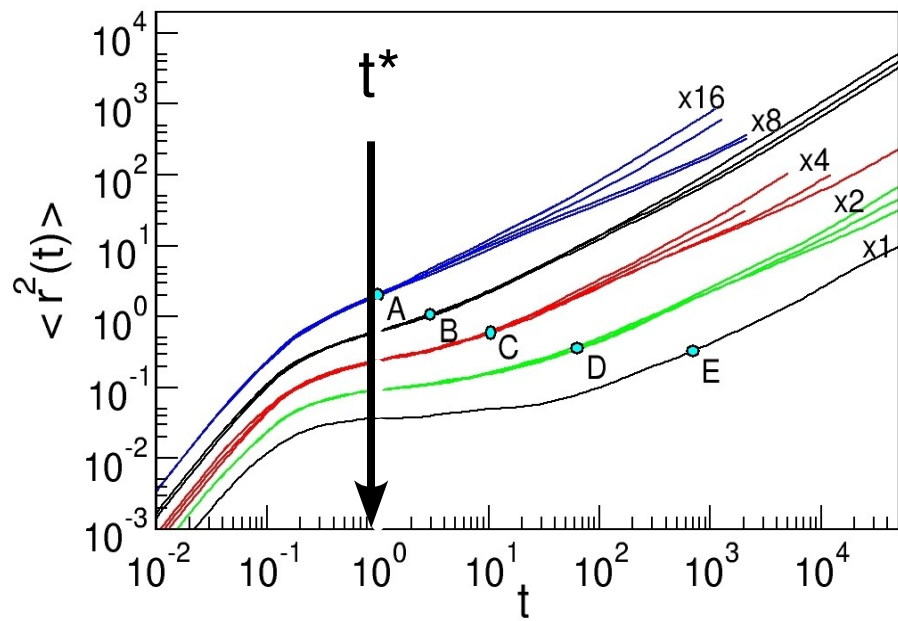
# A tag for MSDs

Vibrational dynamics is tagged by square amplitude of vibrations:

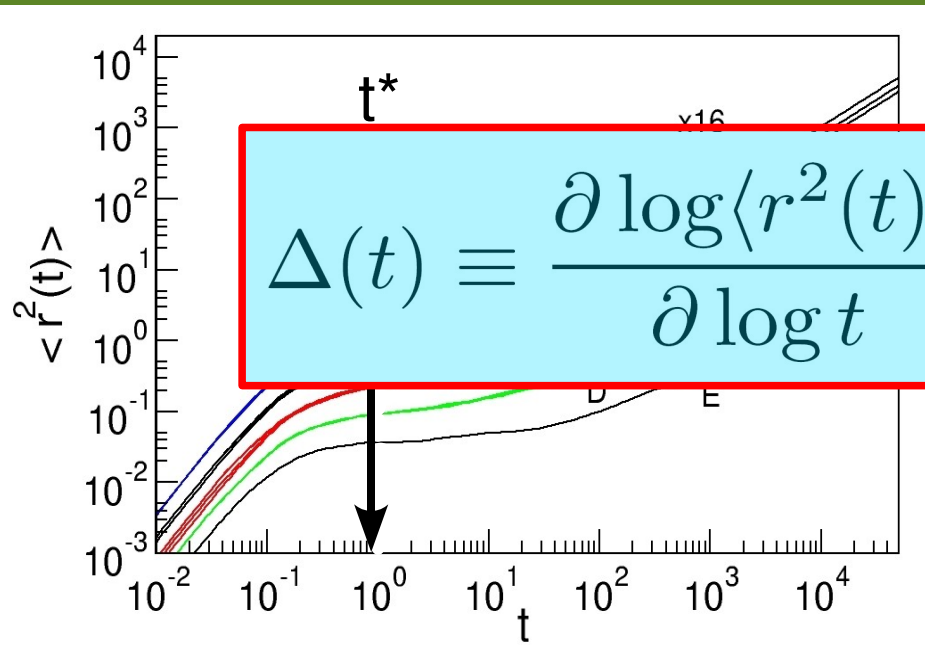
$$\langle u^2 \rangle = \langle \Delta r^2(t = t^*) \rangle$$



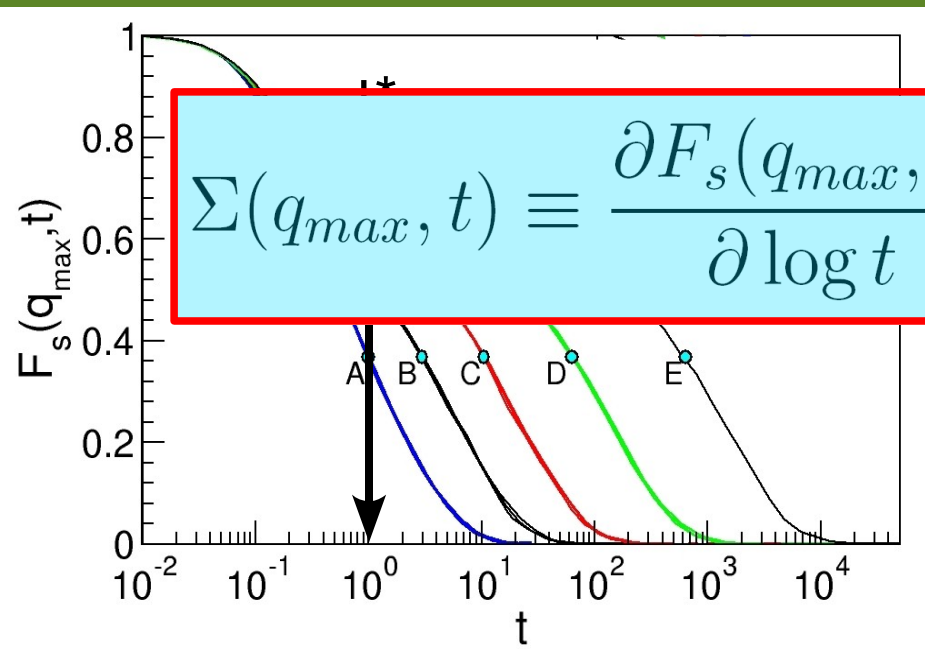
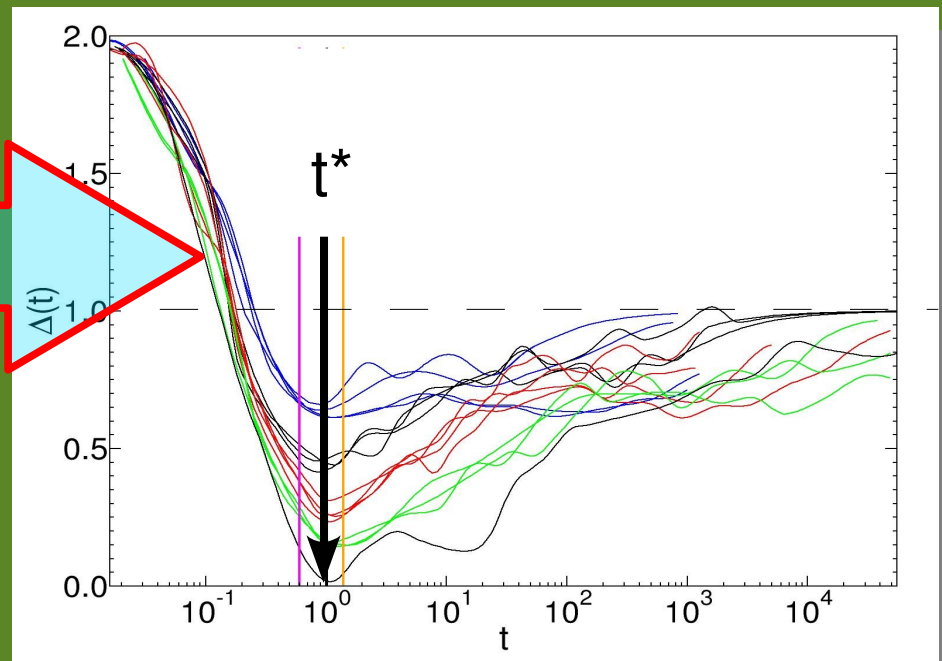
# Inflection points in MDS and ISF coincide



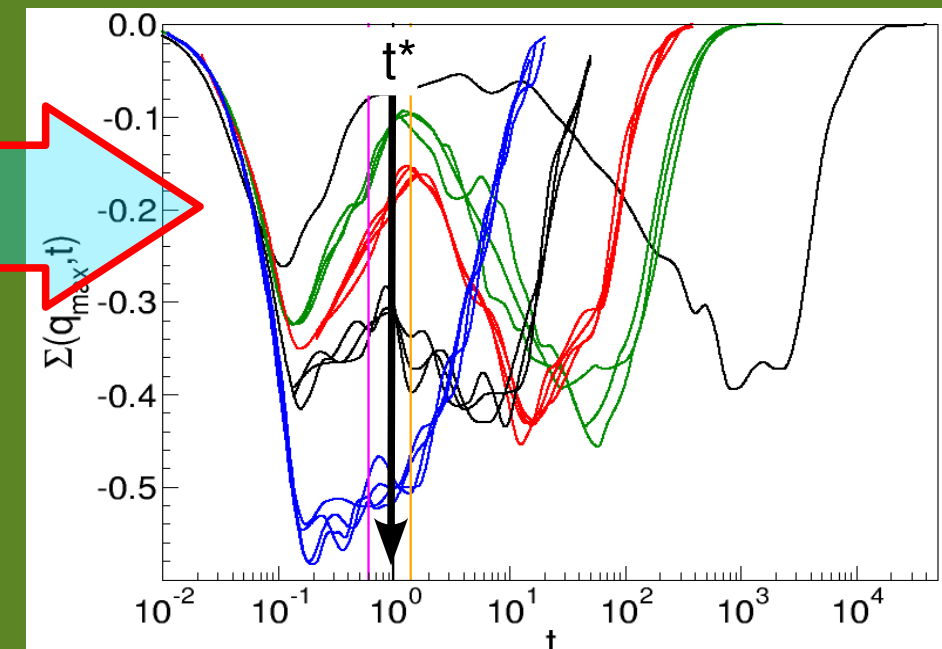
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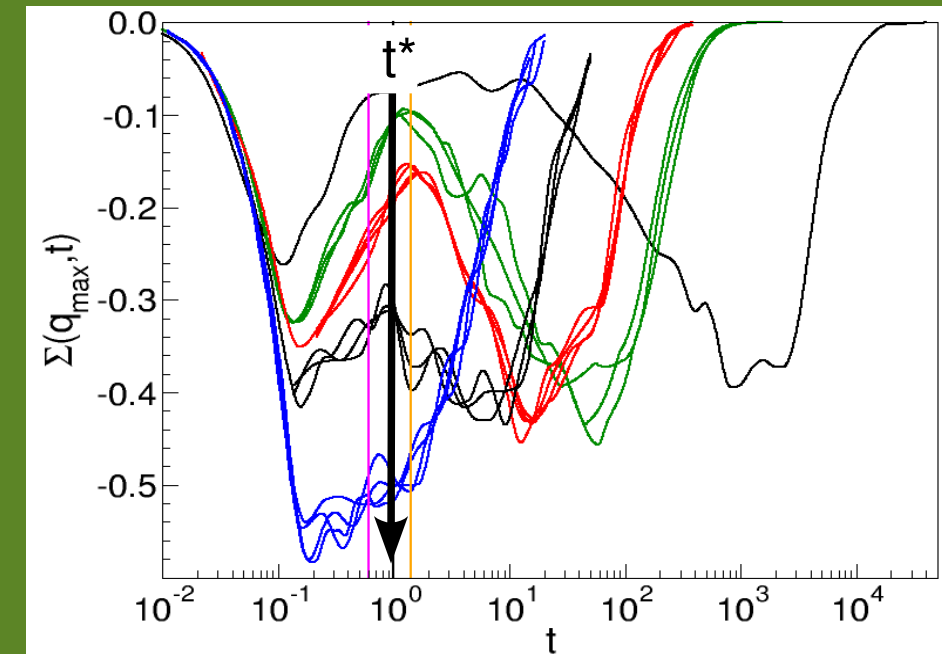
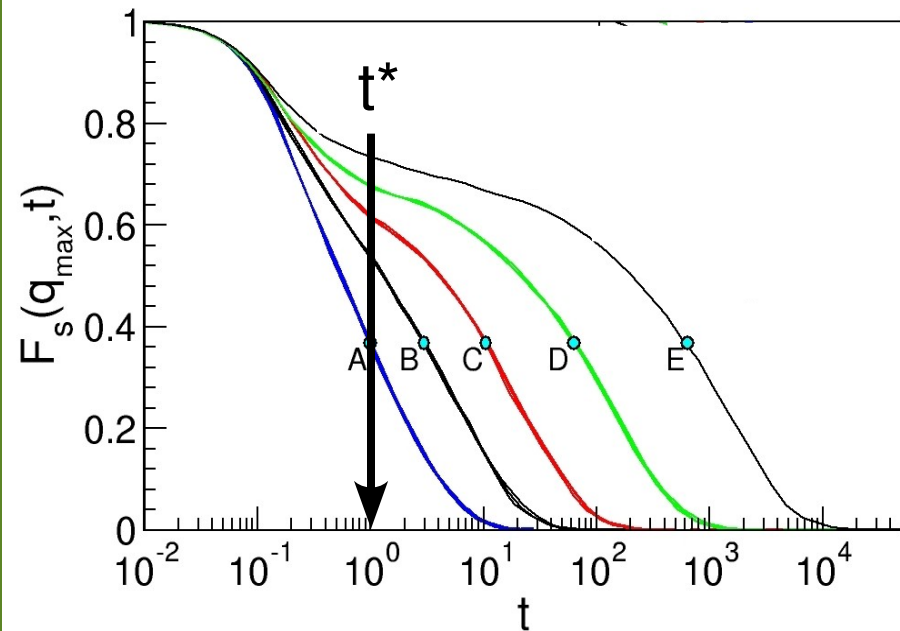
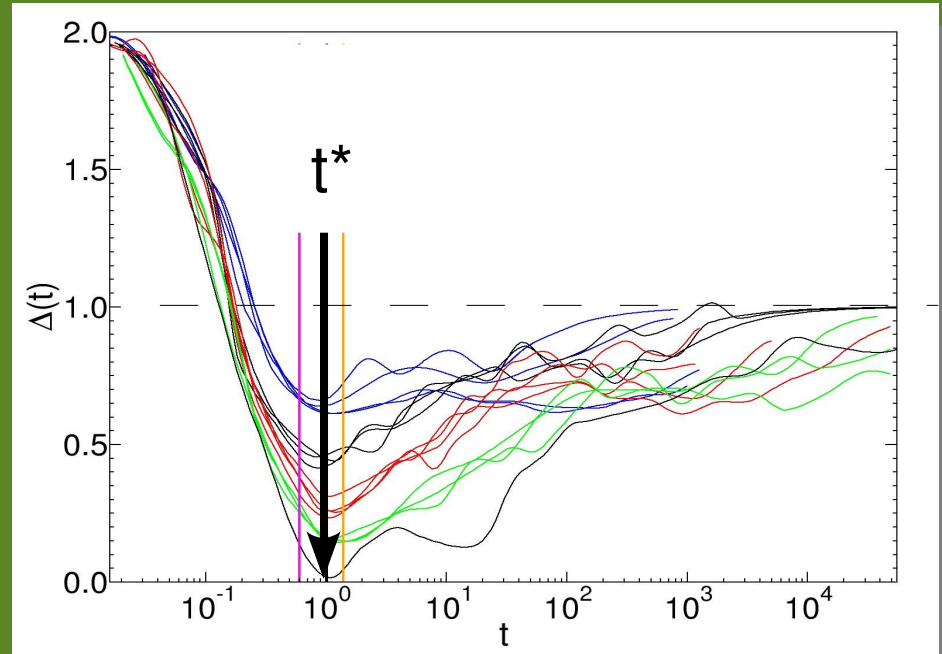
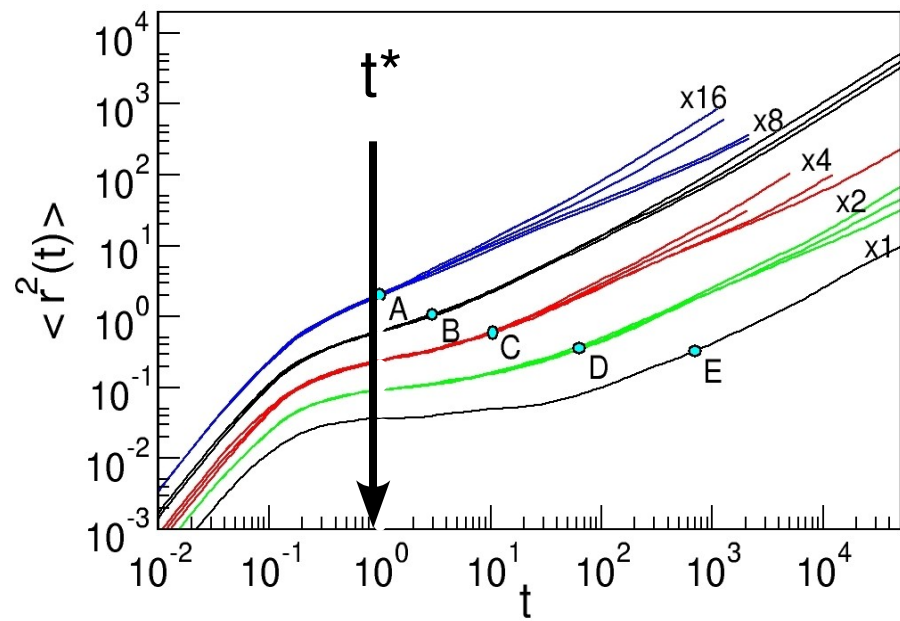
$$\Delta(t) \equiv \frac{\partial \log \langle r^2(t) \rangle}{\partial \log t}$$



$$\Sigma(q_{max}, t) \equiv \frac{\partial F_s(q_{max}, t)}{\partial \log t}$$

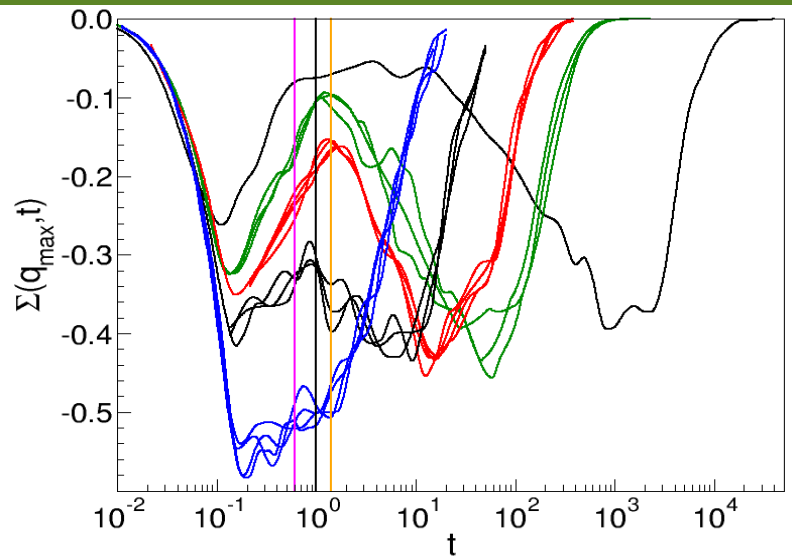


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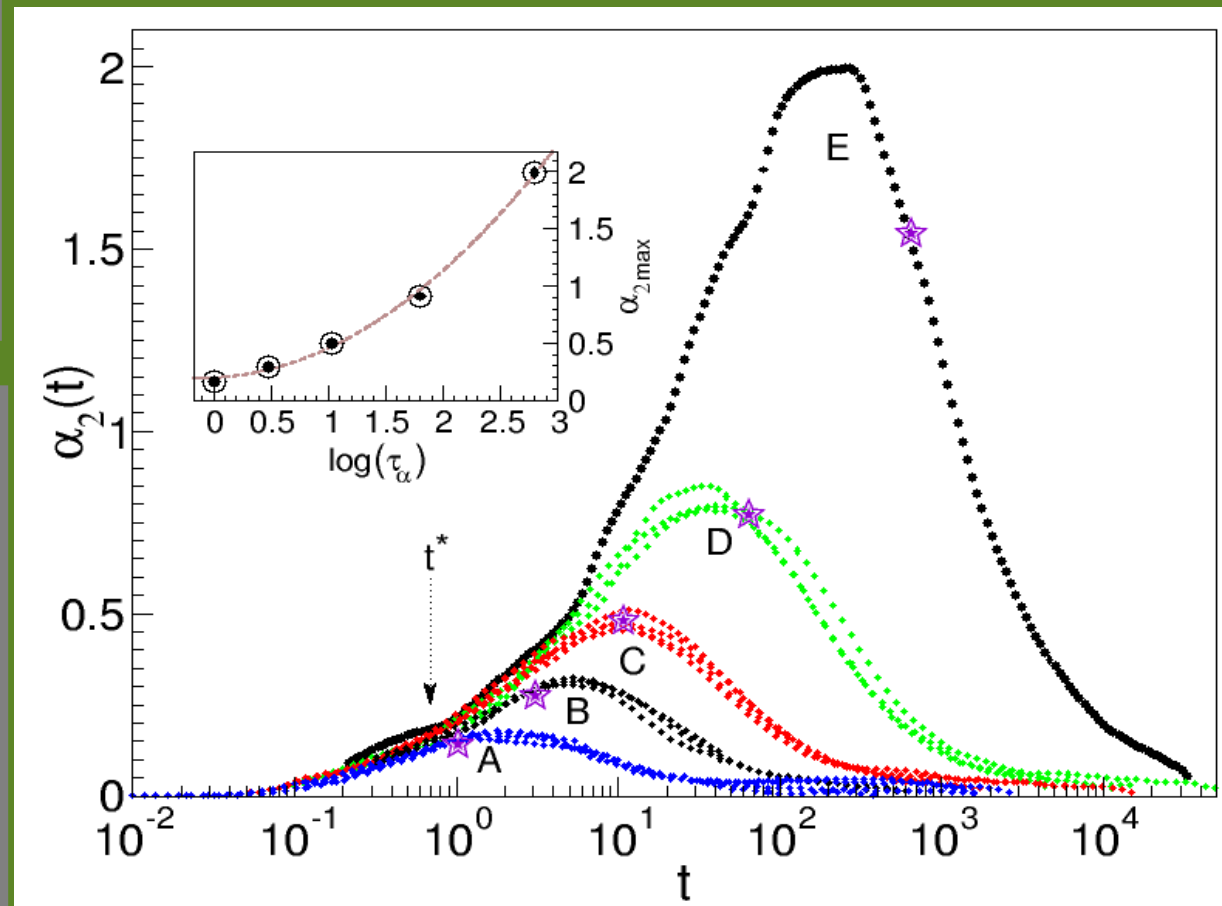
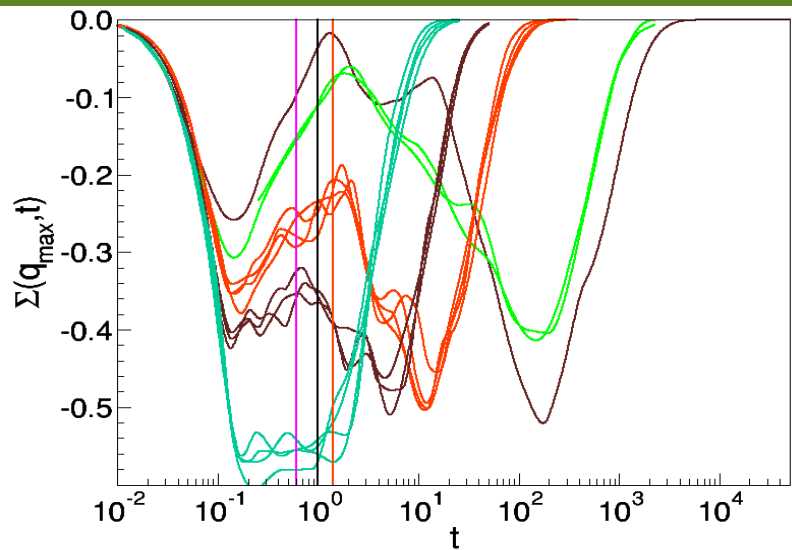


# Heterogeneous dynamics growth

Heterogeneous Dynamics **still negligible at  $t^*$**   
and modify strongly the structural relaxation **only after**



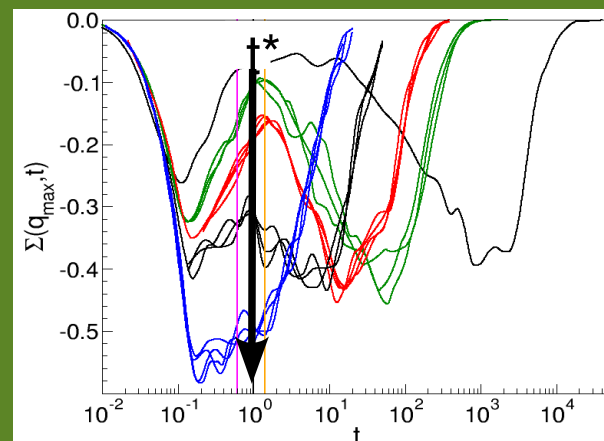
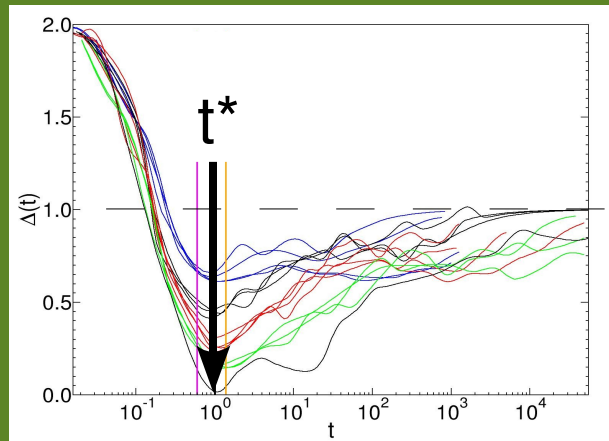
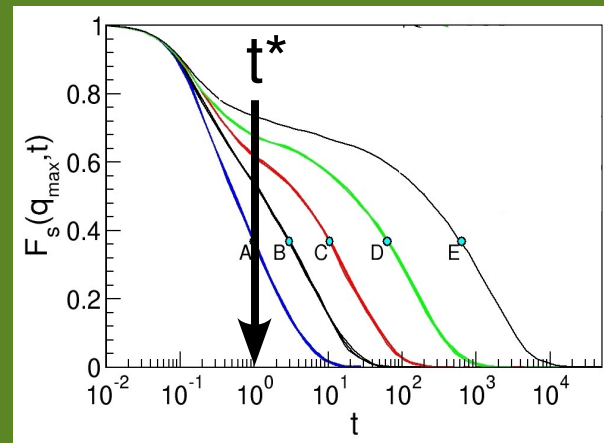
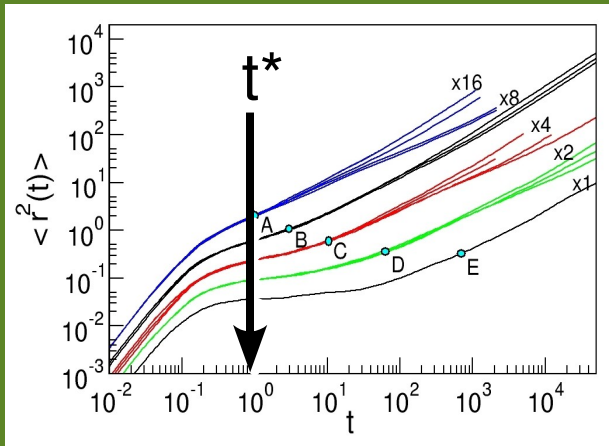
$$\alpha_2(t) = \frac{3}{5} \frac{\langle r^4(t) \rangle}{\langle r^2(t) \rangle^2} - 1$$



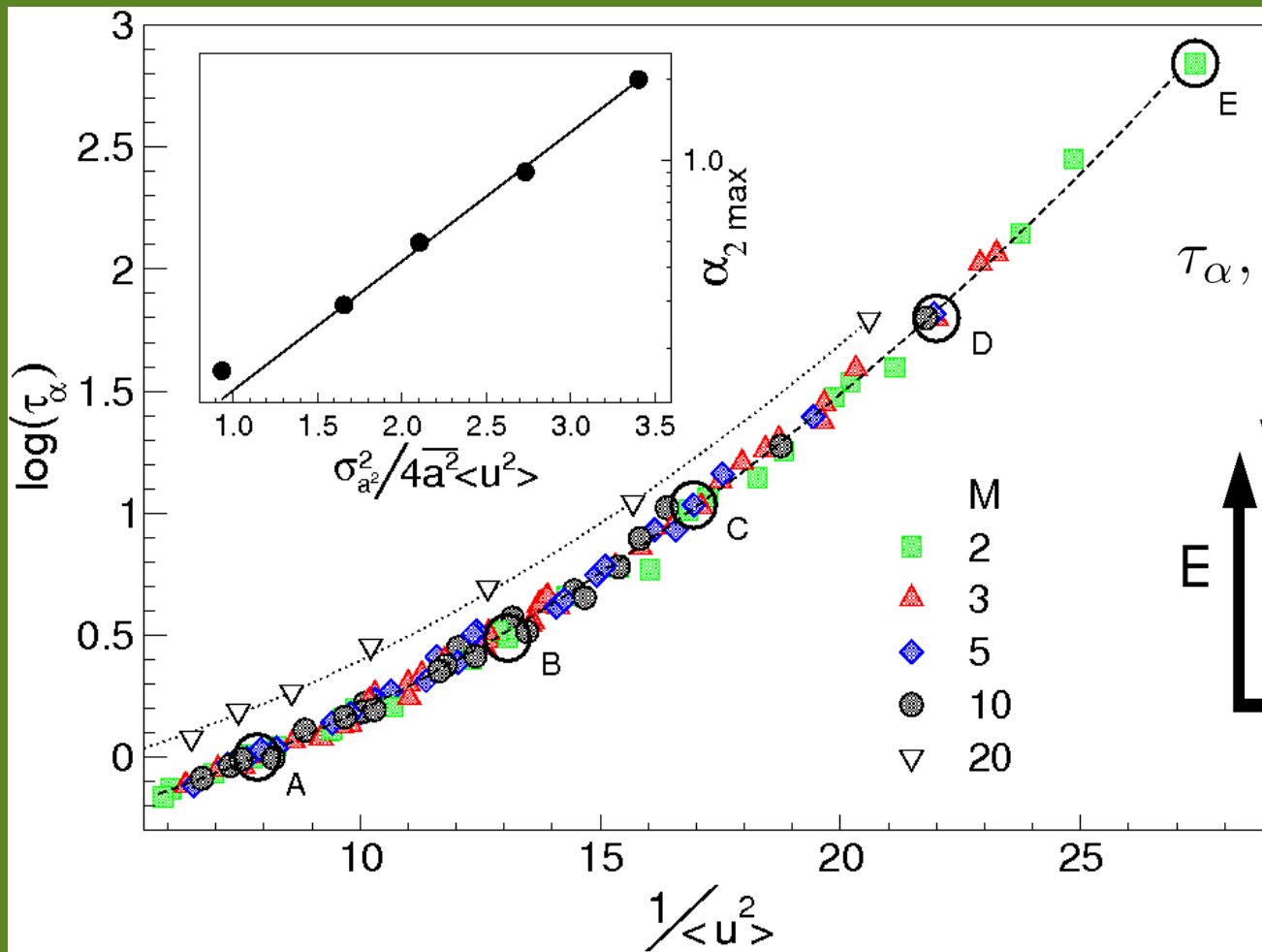
# No differences with experimental conventional definitions

$$\langle u_{MSD}^2 \rangle = \langle \Delta r^2(t = t^*) \rangle$$

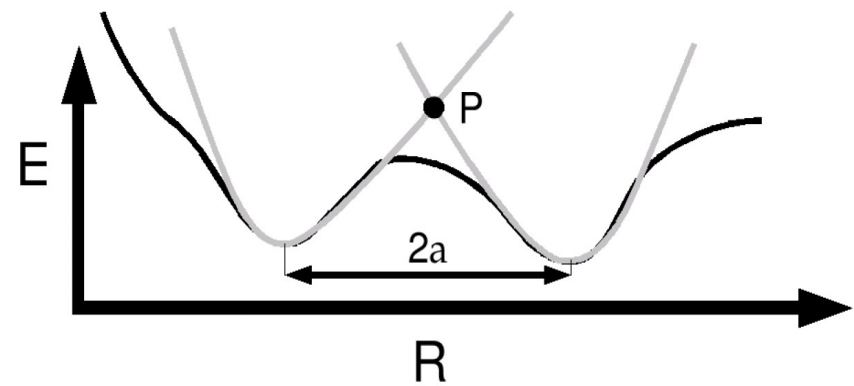
$$\langle u_{ISF}^2 \rangle = -\frac{3}{q_{max}^2} \ln [F_s(q_{max}, t^*)]$$



# “Universal” scaling curve (poly+BM)



$$\tau_\alpha, \eta \propto \exp \left( \frac{a^2}{2\langle u^2 \rangle} + \frac{\sigma_{a^2}^2}{8\langle u^2 \rangle^2} \right)$$



Quartic term due to the spread of the distance between the EL minima (heterogeneous dynamics)

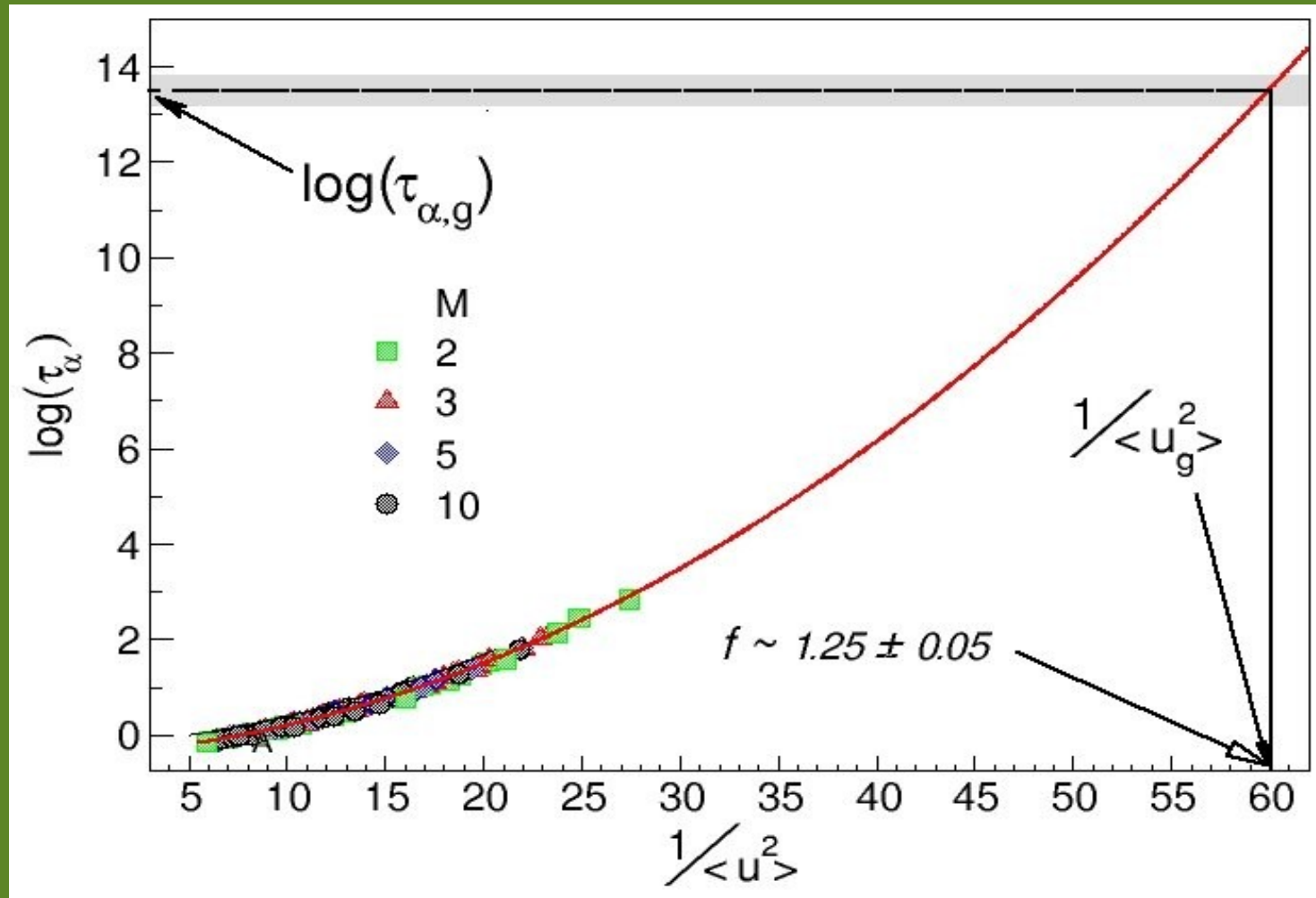
**No evidence of dynamic divergencies (Dyre et al., 2008)**

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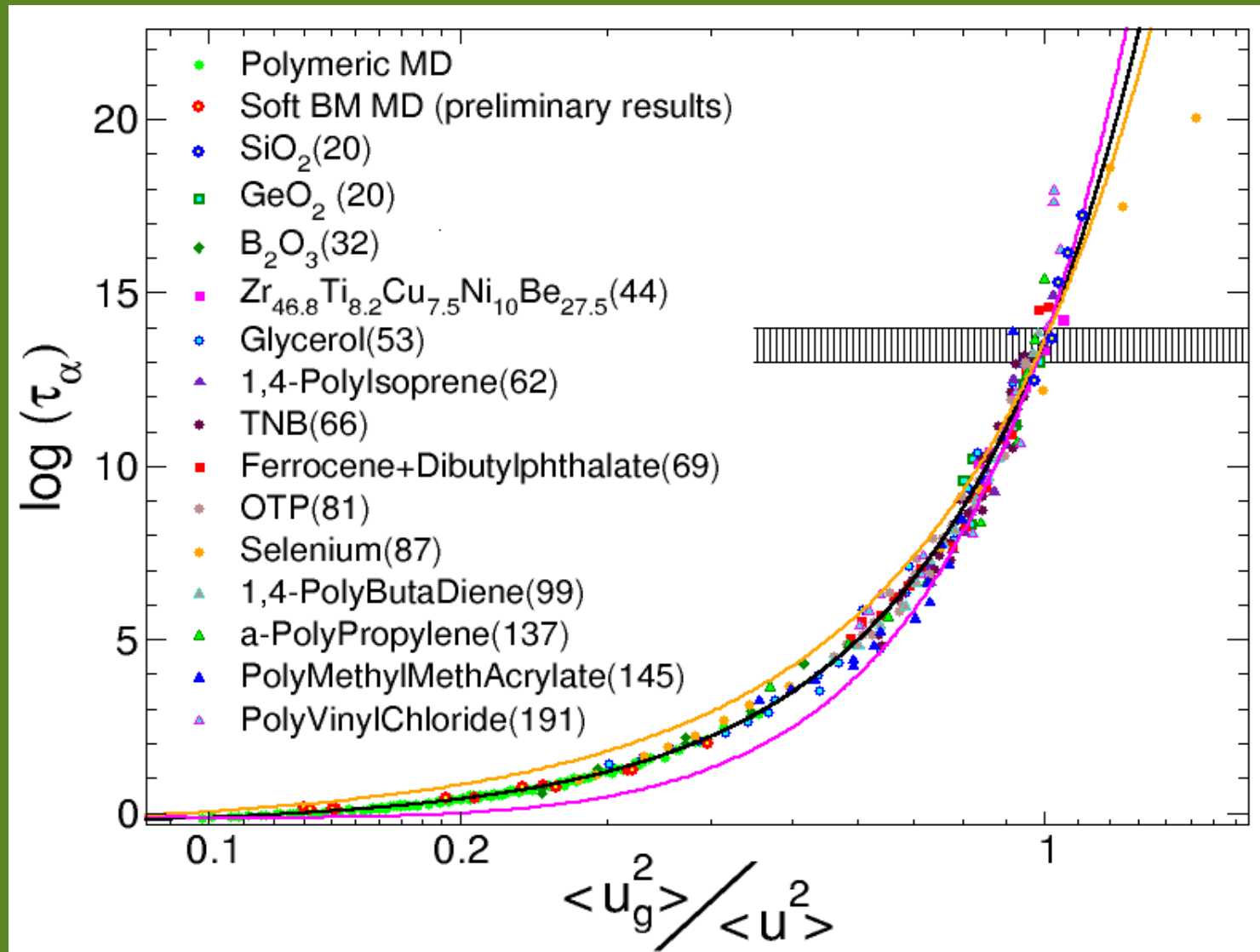
# Locating the Glass Transition of the MD models



$\langle u_g^2 \rangle$  consistent with both the  
Lindemann ratio  $f$  and free-volume ideas

1 MD time unit:  $\approx 1-10$  ps

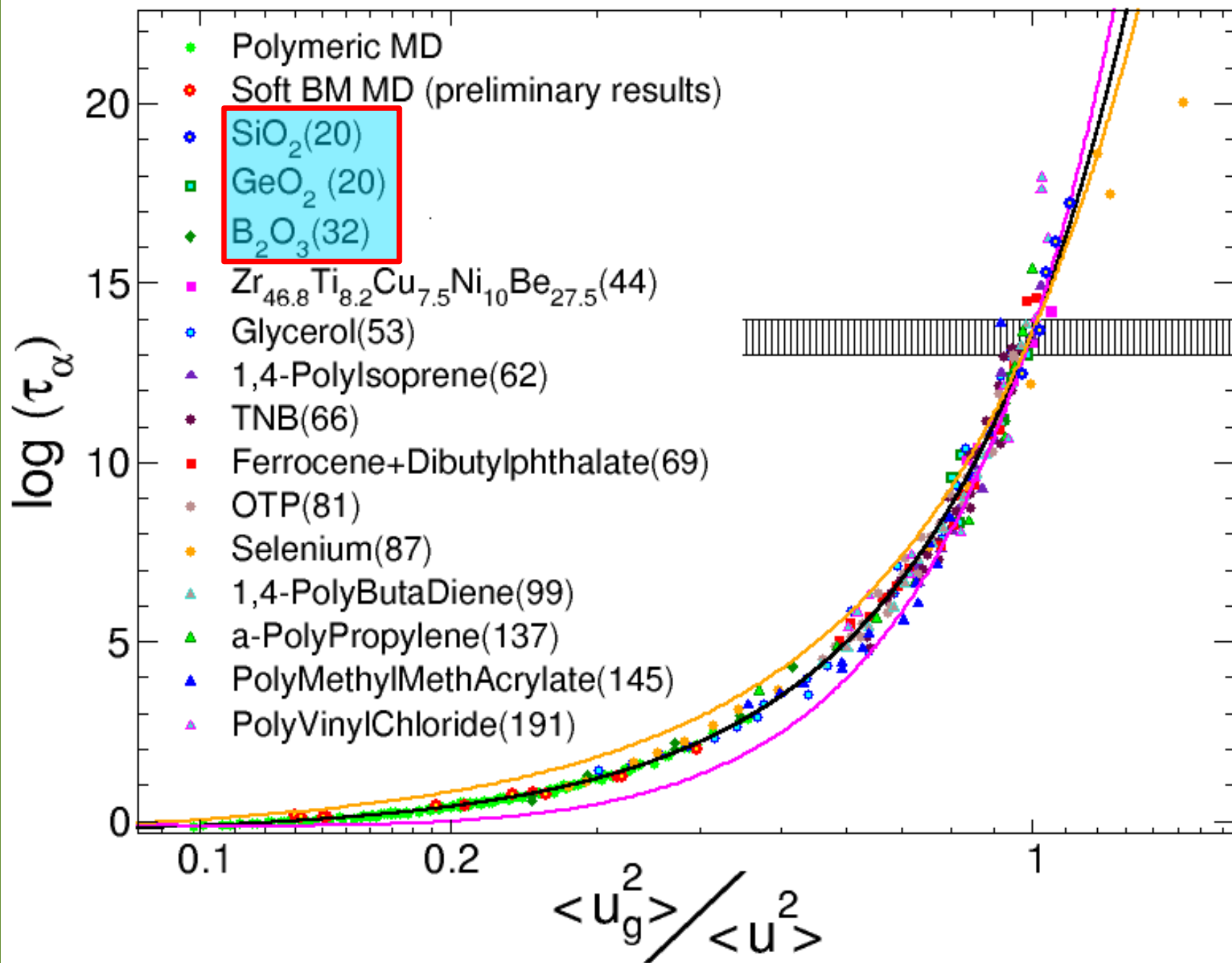
# Rescaled universal curve



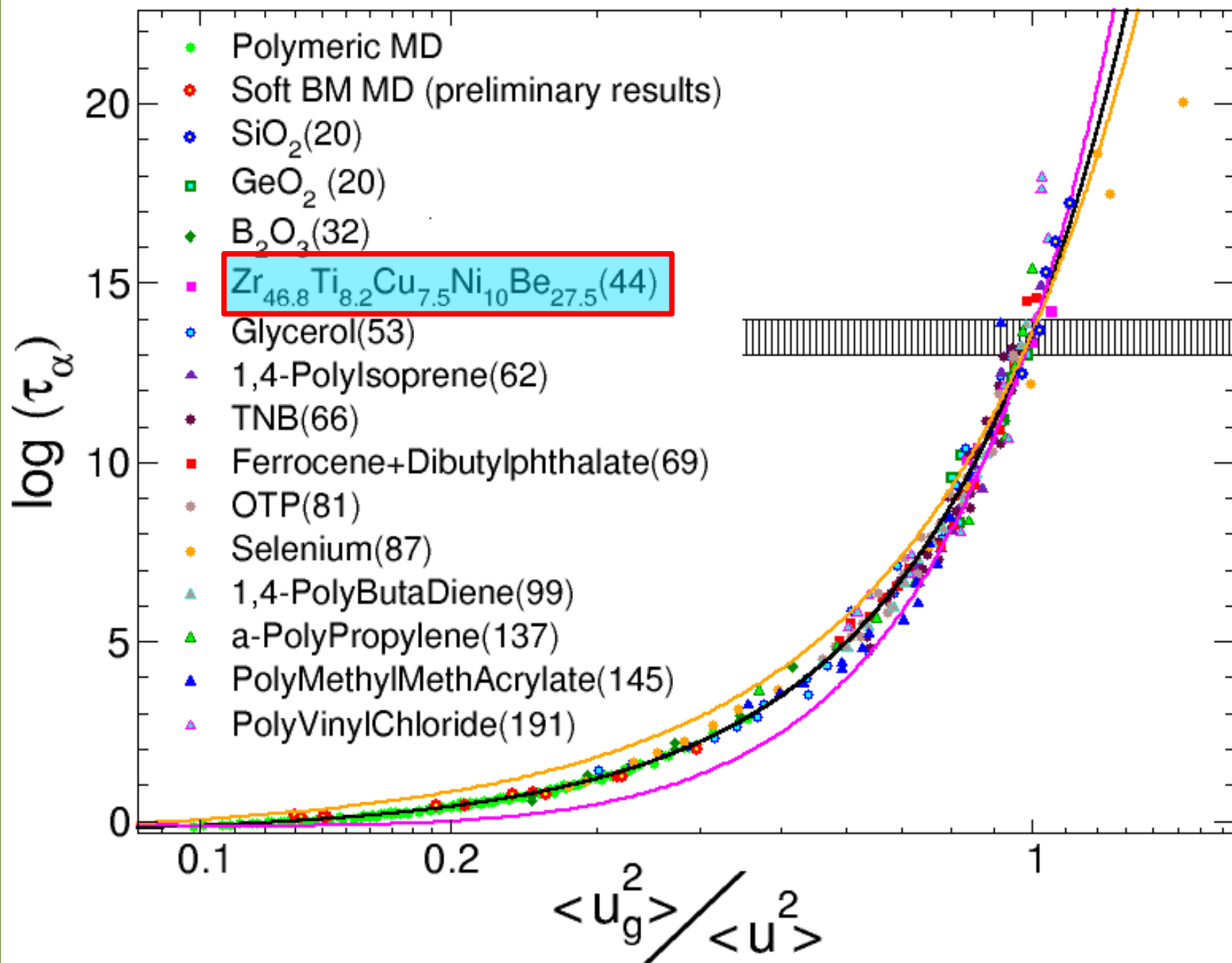
The reduced MD master curve is suited for comparison with experimental data.

Agreement over about **eighteen decades** of relaxation times.

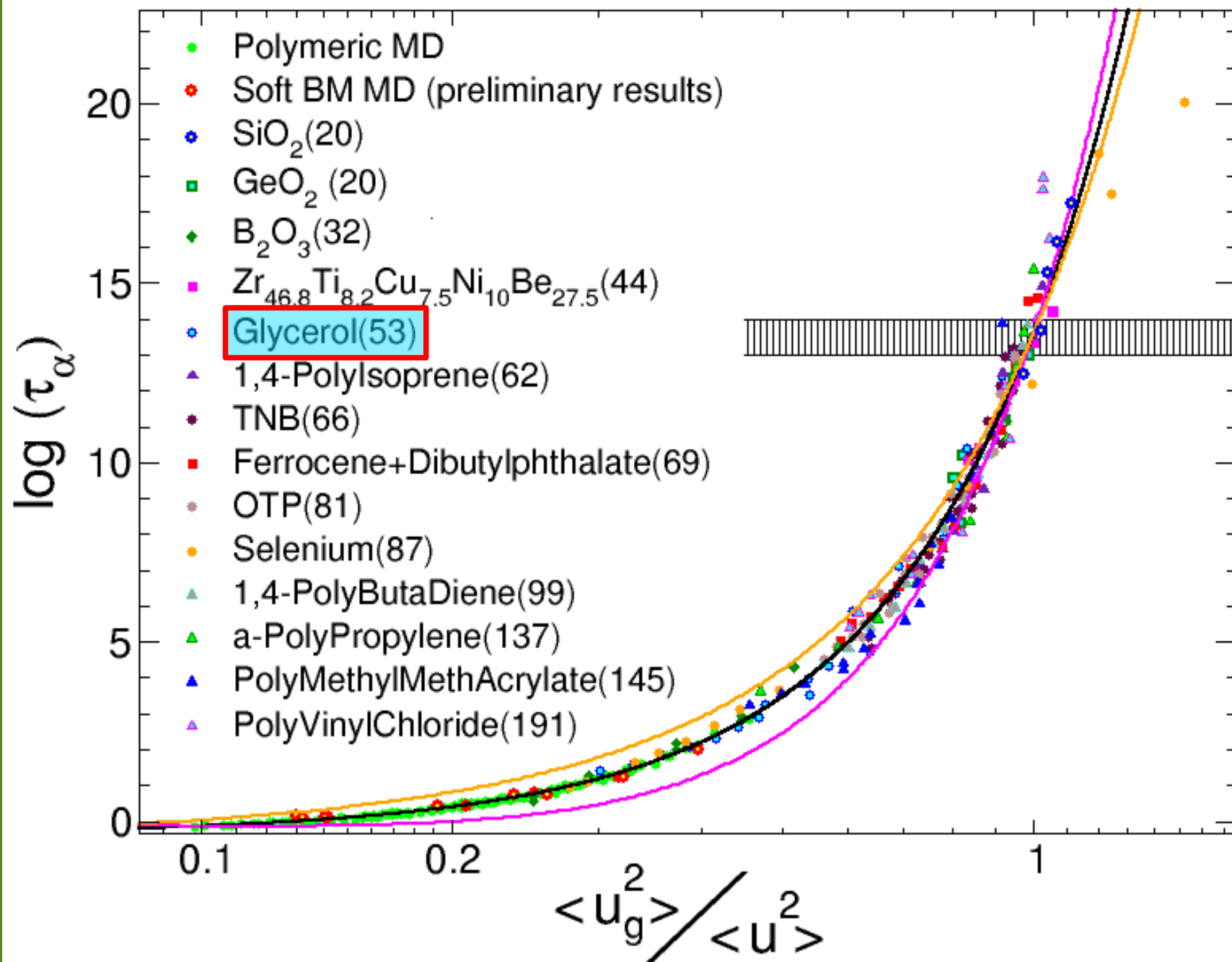
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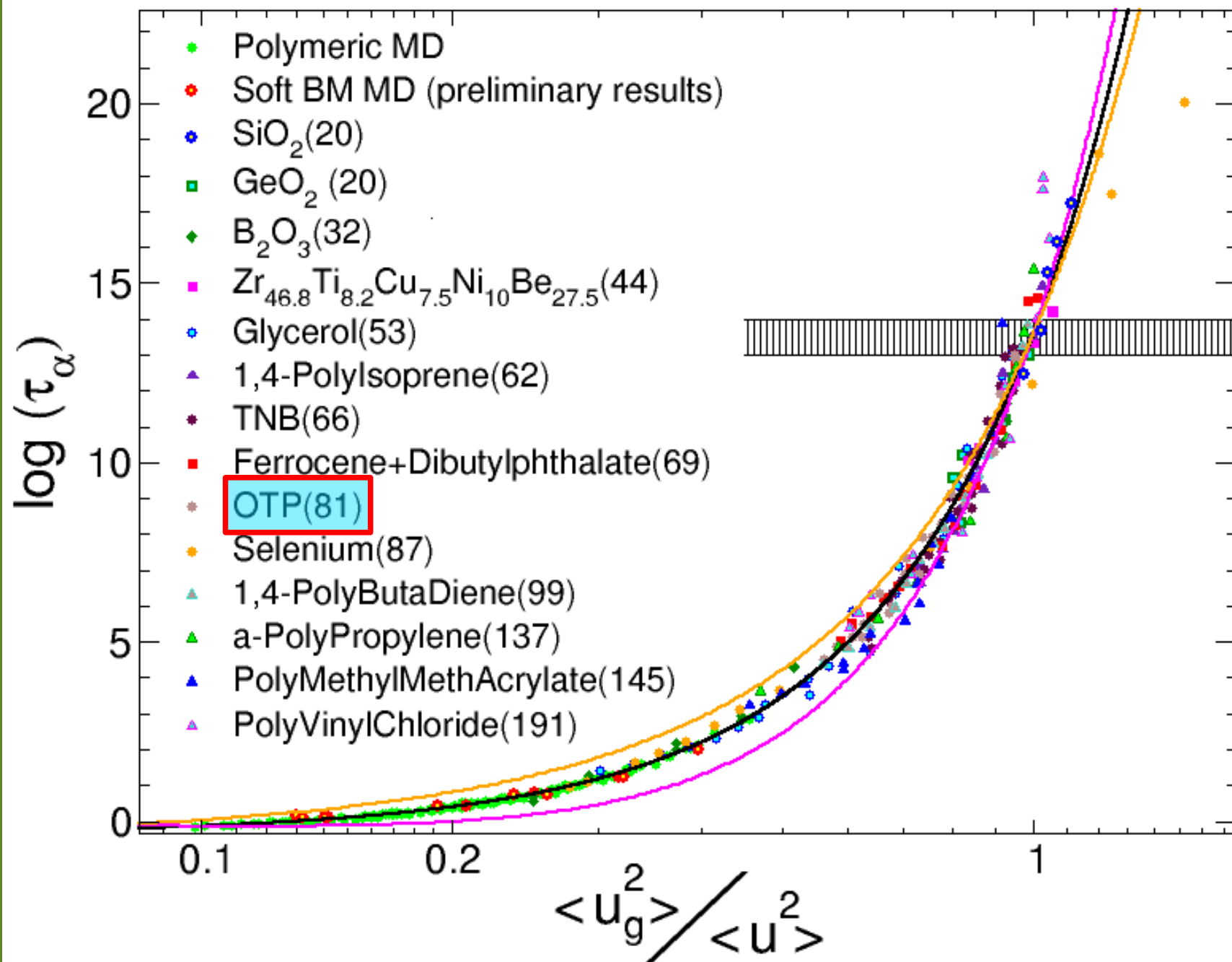
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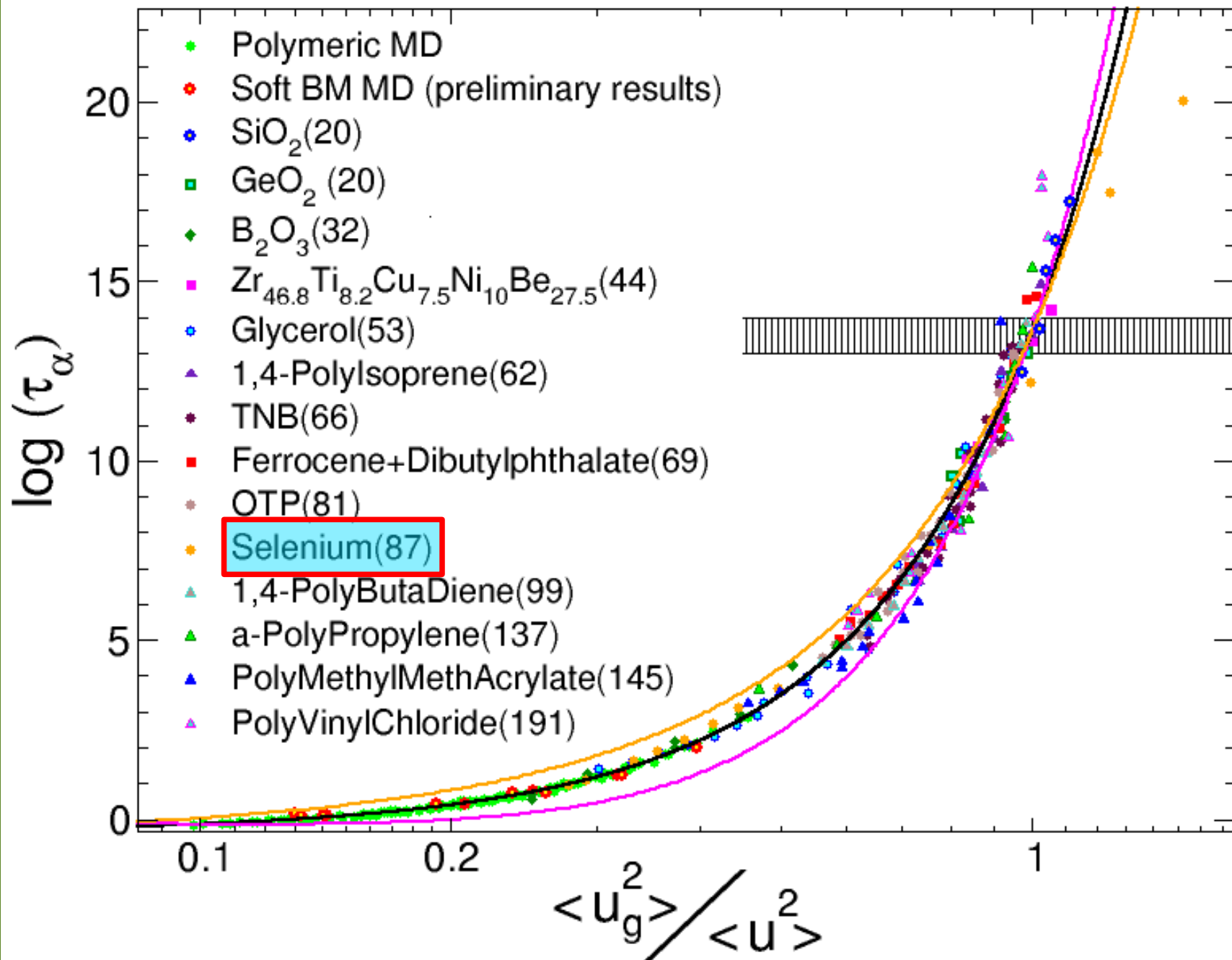
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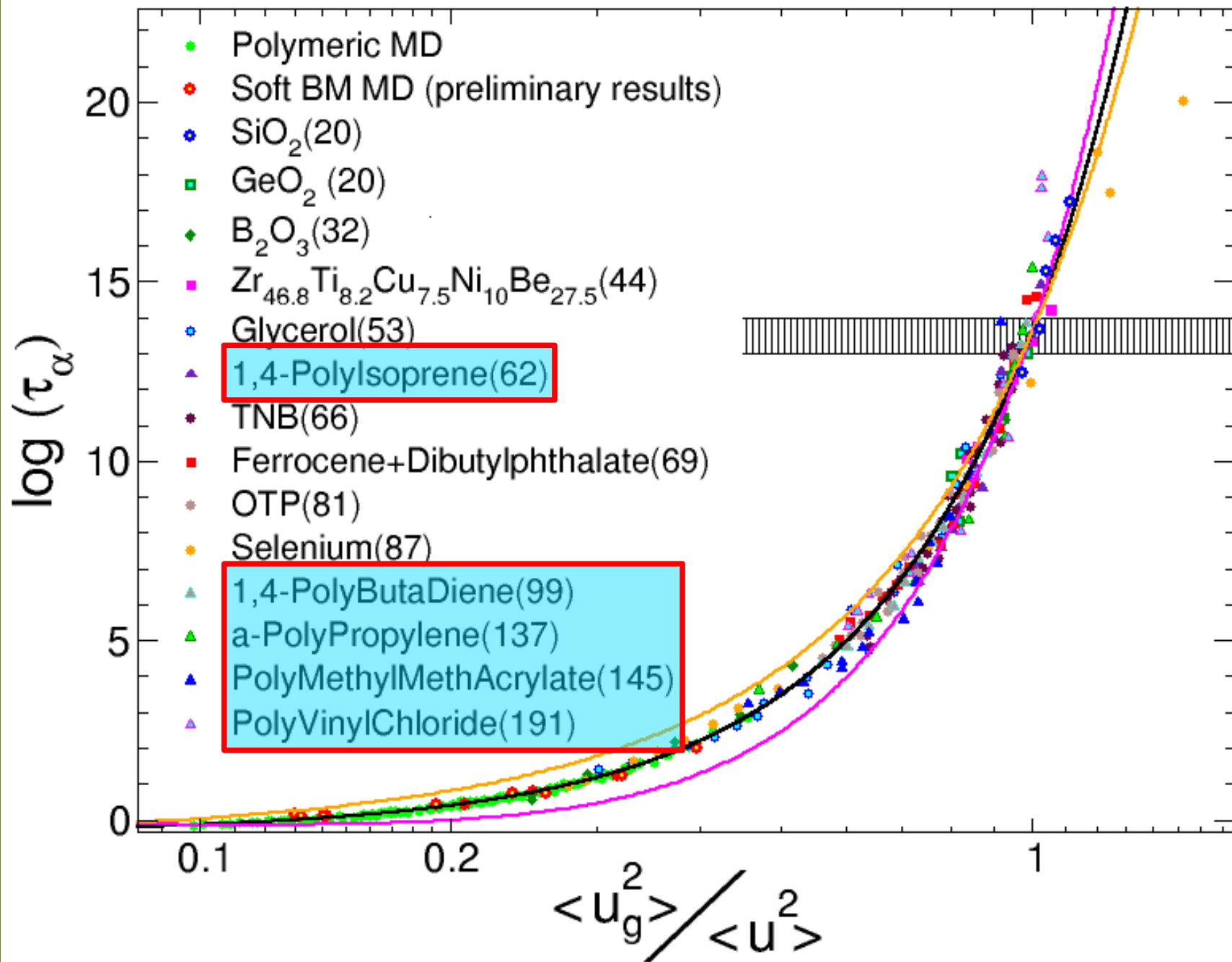
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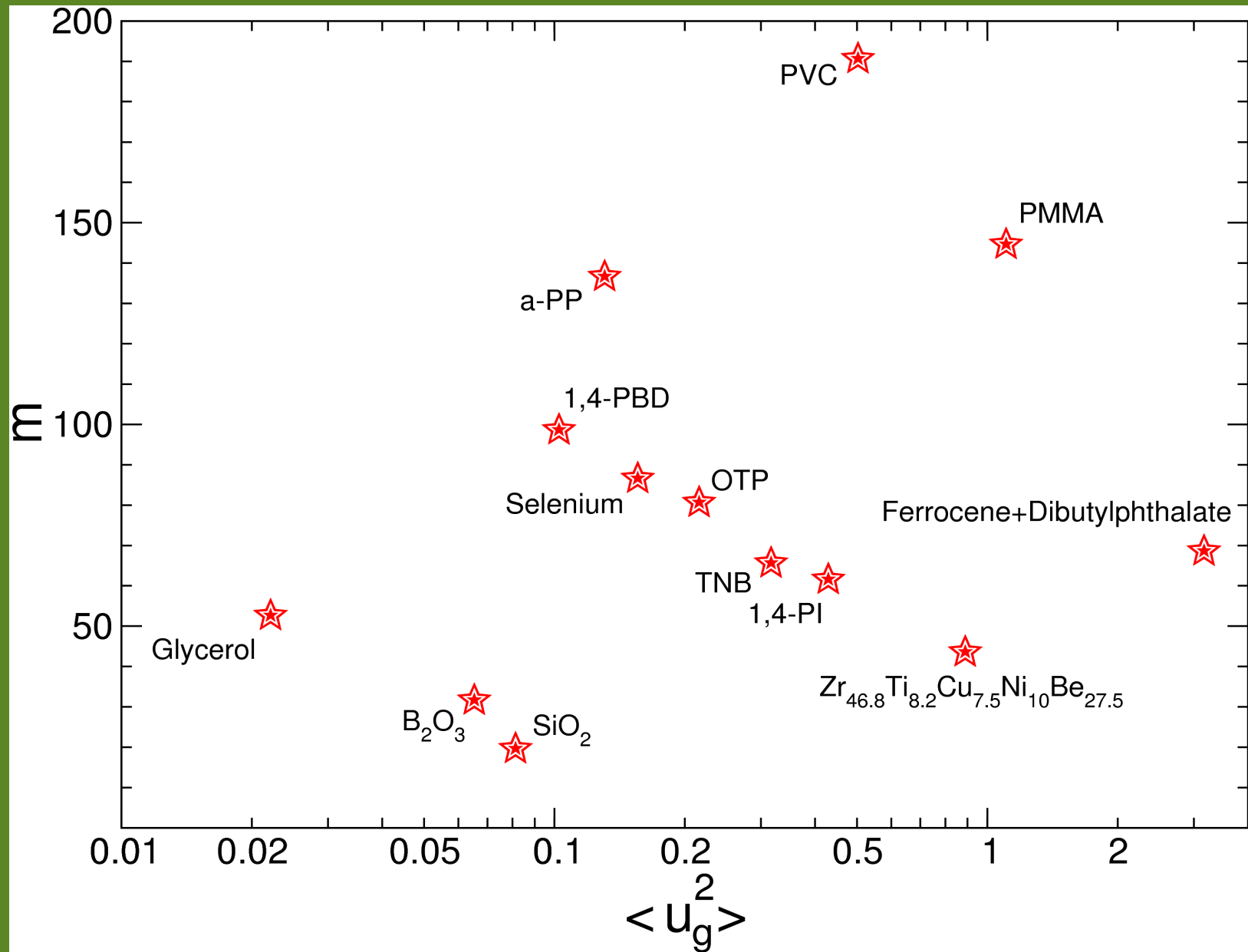


# Rescaled universal curve





# No correlation between fragility and $\langle u_g^2 \rangle$



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- Agreement with the available experiments over about **eighteen decades** of relaxation times and a **large fragility range** (  $20 \leq m \leq 190$  ).

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- **No** evidence of dynamic **divergencies**.
- Agreement with the available experiments over about **eighteen decades** of relaxation times and a **large fragility range** (  $20 \leq m \leq 190$  ).
- Consistency with the **Lindemann melting criterion** and support for the **iso free-volume** interpretation of the glass transition.