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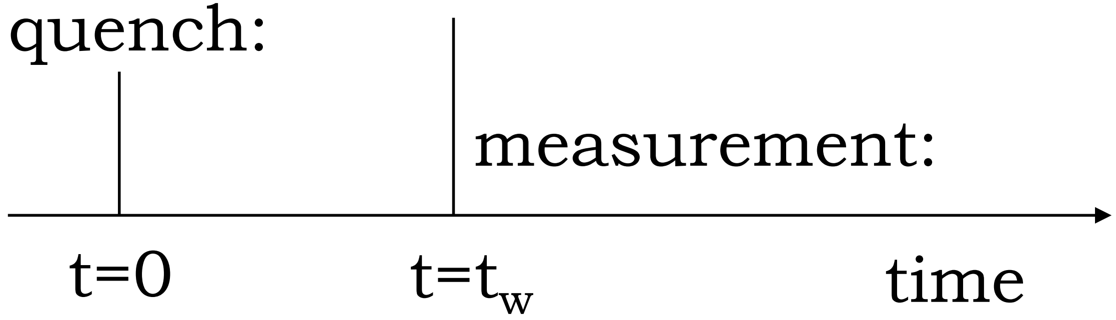
Memory effects in the relaxation of the energy master equation

- Aging - Kovacs effect
- Gaussian trap model
- Conclusions

GD, Andreas Heuer

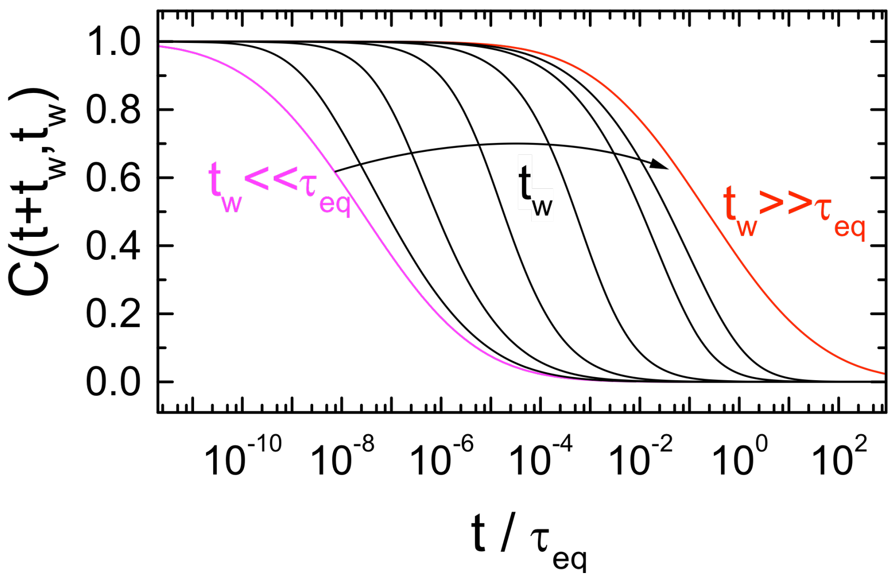
Aging – Kovacs effect:

aging exp.:



$$C(t, t_w) = \langle M(t) M(t_w) \rangle$$

two-time quantity



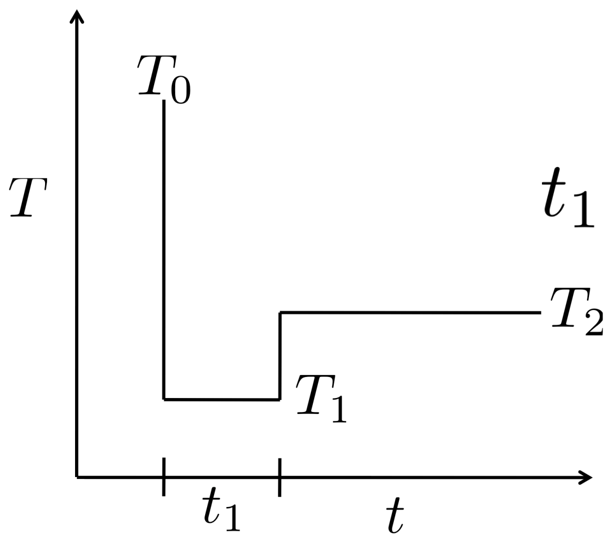
information about
- FDT-violations (effective temp)
- nonlinear relaxation
- internal clock
- …

Aging – Kovacs effect - contd:

thermodynamic variables: one-time quantities

can the relaxation of the volume be described by one external parameter (temperature)?

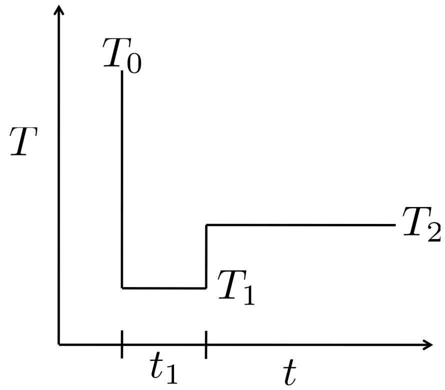
idea (Kovacs 1963):

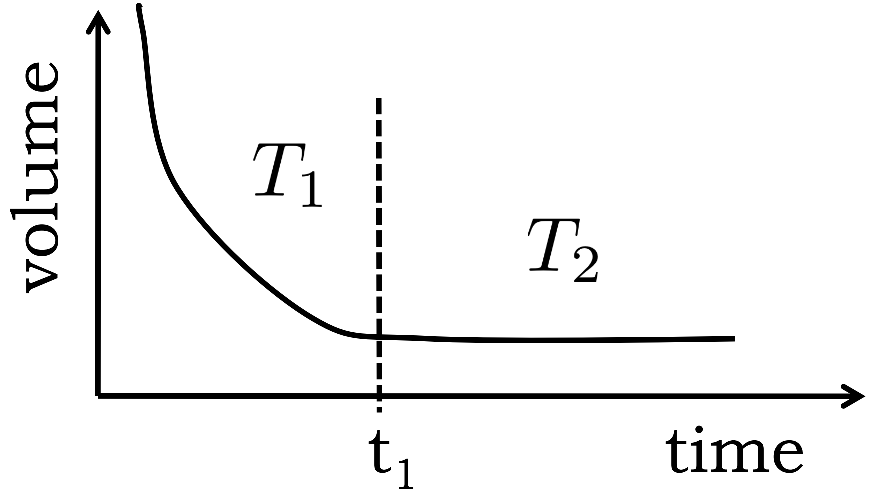


$$t_1 : V(t_1, T_1) = V_{eq}(T_2)$$

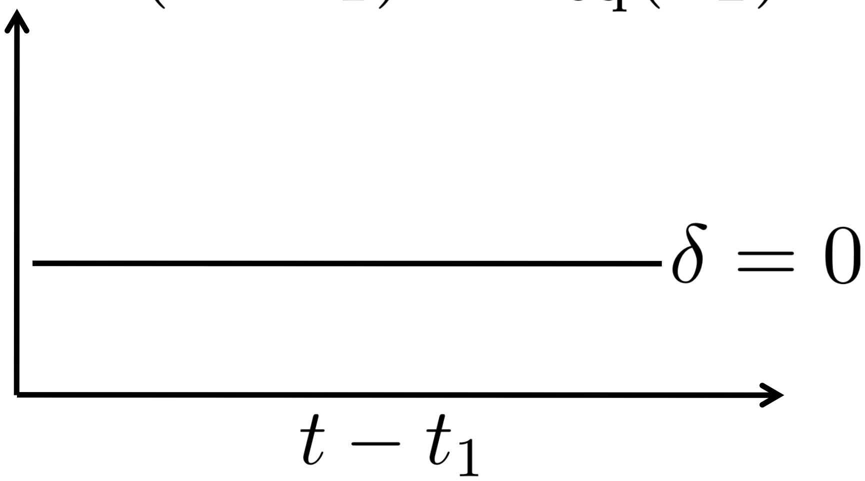
Aging – Kovacs effect - contd:

expectation:



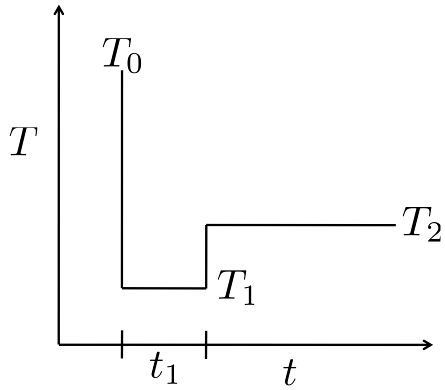


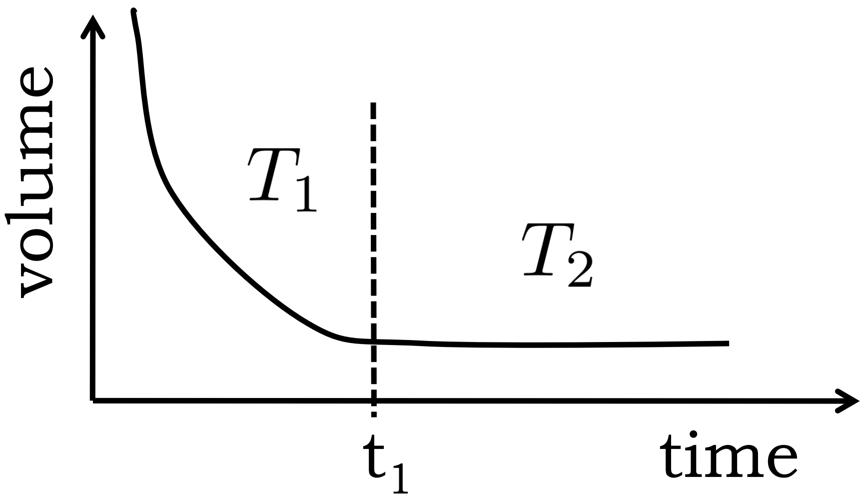
$$\delta = V(t - t_1) - V_{\mathrm{eq}}(T_2)$$



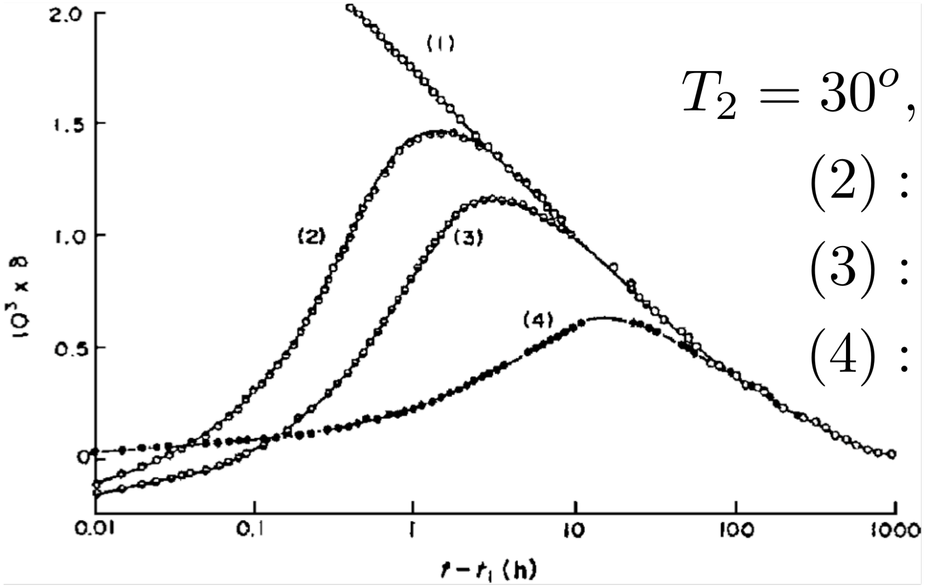
Aging – Kovacs effect - contd:

expectation:





result:
Kovacs 1963
(PVC)



$T_2 = 30^o, T_0 = 40^o$

$(2) : T_1 = 10^o$

$(3) : T_1 = 15^o$

$(4) : T_1 = 25^o$

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Aging – Kovacs effect - contd:

many (experiments and) model calculations:

- Berthier, Bouchaud 2002: Ising spin glass
- Bertin et al. 2003: domain growth, exp. trap model
- Buhot 2003: kinetically constrained models
- Cugliandolo et al. 2004: p-spin models
- Arenzon, Sellitto 2004: facilitated spin models
- Mossa, Sciortino 2004: oTP -model
- Aquino et al. 2008: two level systems

qualitatively:
distribution of relaxation times or length scales
explains the Kovacs hump

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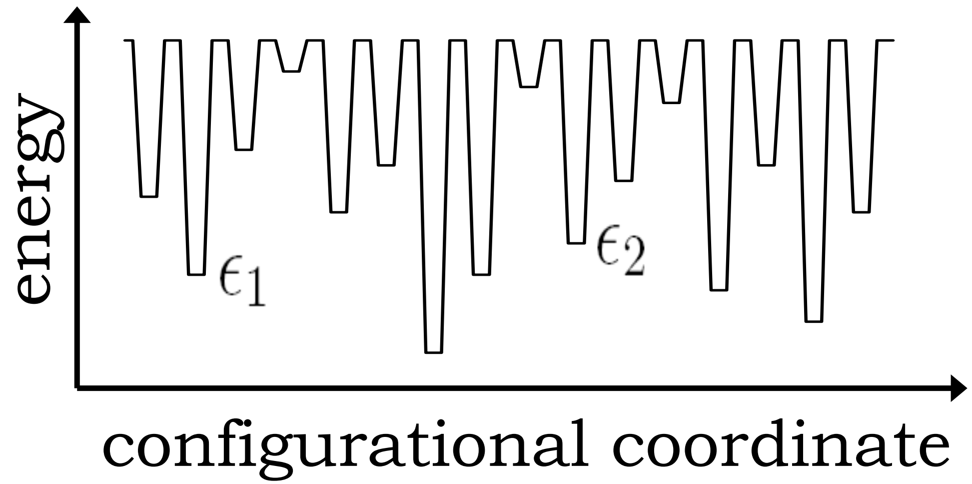
trap models:

Jeppe Dyre (1995) !
Monthus, Bouchaud (1996)



distribution: $\rho(\epsilon) \sim e^{-\epsilon^2/2\sigma^2}$

(or exponential distribution)

transition rates:

$$W(\epsilon_{\rm fin}|\epsilon_{\rm in}) = \boxed{\rho(\epsilon_{\rm fin})} \times \boxed{e^{\epsilon_{\rm in}/T}}$$

random choice of arrival trap

activated jump out of initial trap

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dynamics (energy master equation):

master equation for populations: $p_T(\epsilon, t)$

$$\dot{p}_T(\epsilon, t) = -e^{\epsilon/T} p_T(\epsilon, t) + \rho(\epsilon) \int d\epsilon' e^{\epsilon'/T} p_T(\epsilon', t)$$

long-time limit: $p_T(\epsilon, t) \sim e^{-(\epsilon - \bar{\epsilon}_T)^2/2\sigma^2}$

(equilibrium) $\bar{\epsilon}_T = -\sigma^2/T$

$$p_T(\epsilon, t) \longrightarrow p_T^{\mathrm{eq}}(\epsilon)$$

temperature jump: $T_0 \longrightarrow T$

$$p_T(\epsilon, t = 0^+) = p_{T_0}^{\mathrm{eq}}(\epsilon)$$

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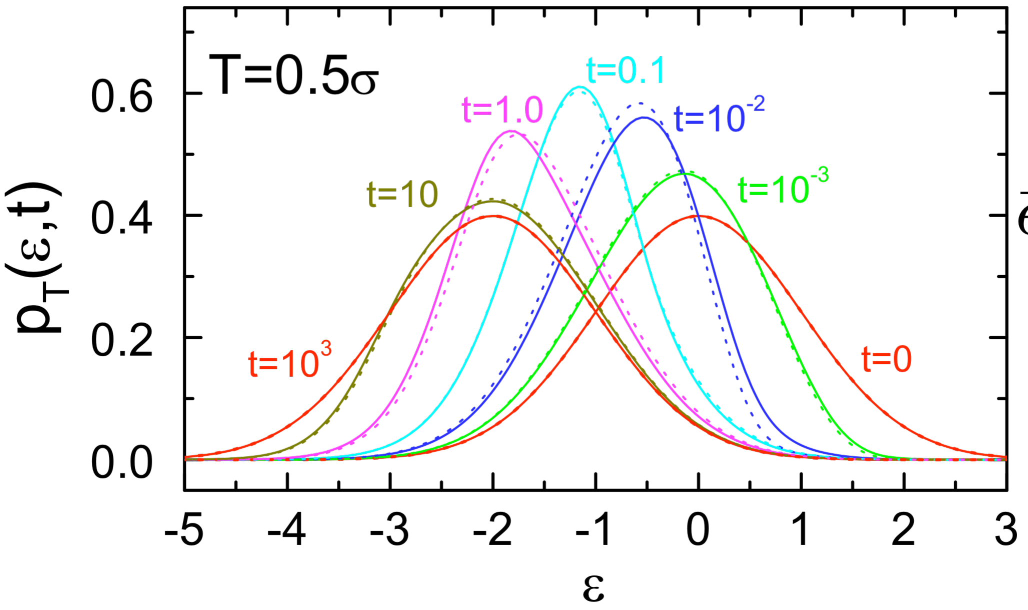
populations after quench: $T_0 = \infty \to T$

propagator: $\dot{G}_T(\epsilon, t|\epsilon') = -e^{\epsilon/T} G_T(\epsilon, t|\epsilon') + \rho(\epsilon) \int d\epsilon'' e^{\epsilon''/T} G_T(\epsilon'', t|\epsilon')$

$$p_T(\epsilon, t) = \int d\epsilon' G_T(\epsilon, t|\epsilon') p_{T_0}(\epsilon')$$



$\bar{\epsilon}_T = -\sigma^2/T$

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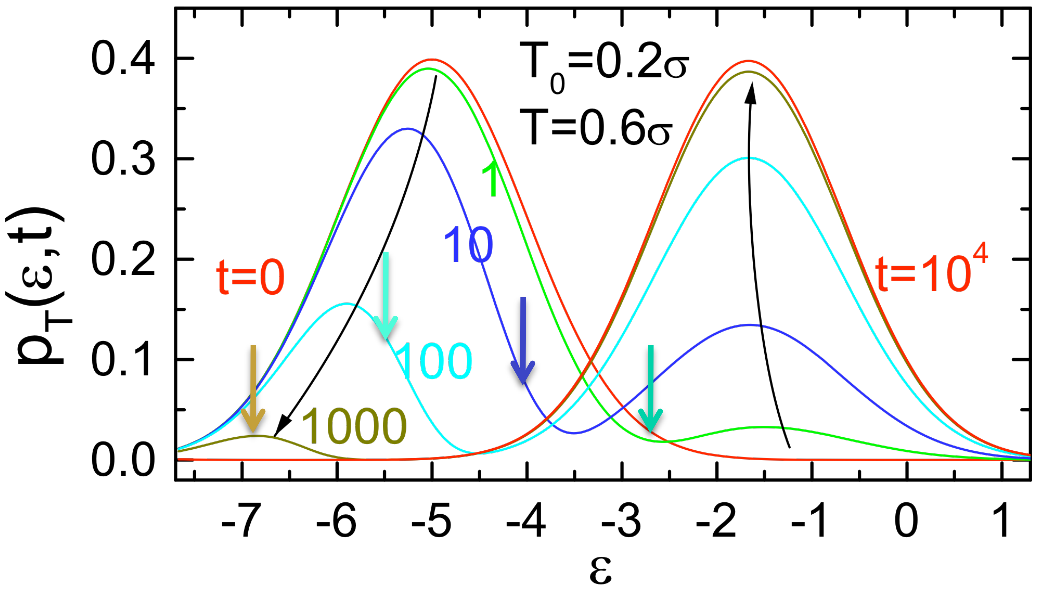
populations after T-jump: $T_0 \to T > T_0$



bimodal distributions!

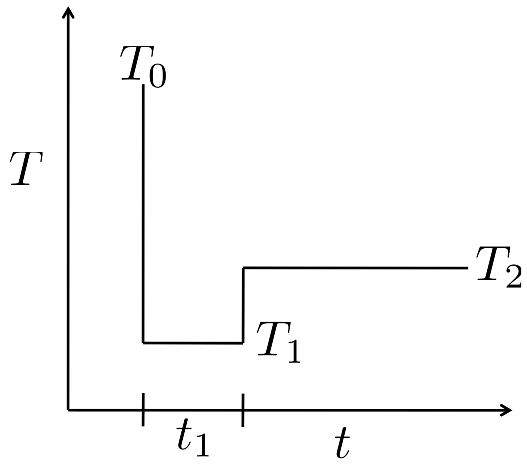
Jeppe Dyre (1995)

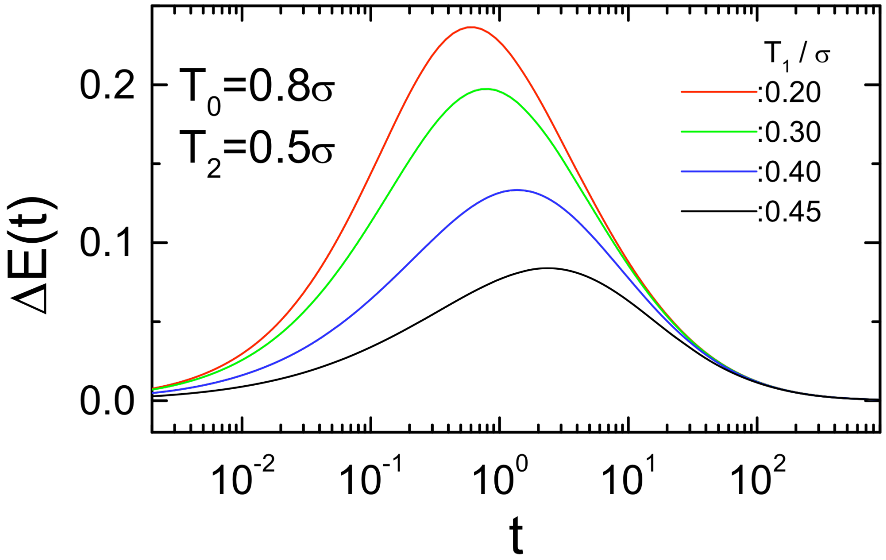
demarcation energy: $\epsilon_D = -T \times \ln(t)$

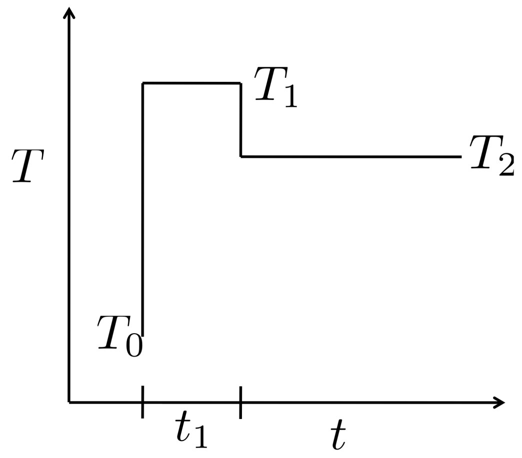
$$\left(e^{\epsilon_D/T} = 1/t\right)$$

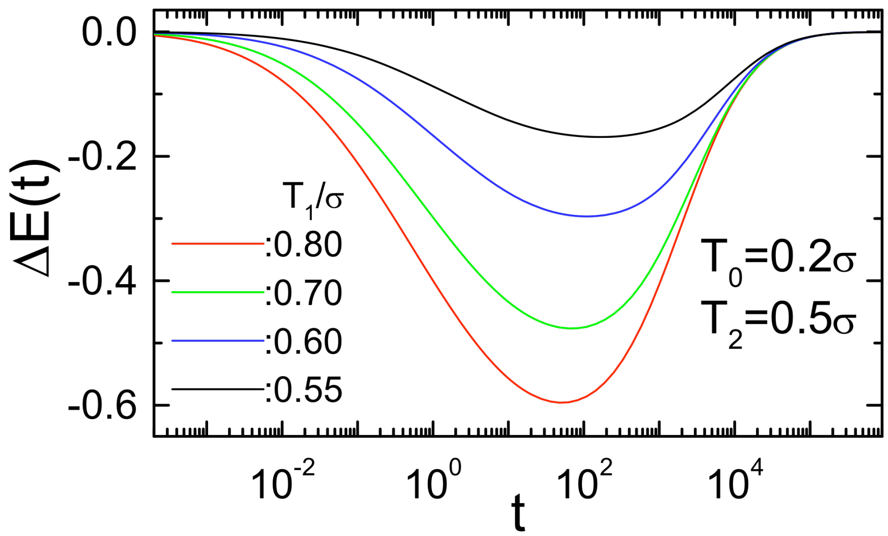
states with energy smaller are 'frozen'

Kovacs hump (energy):

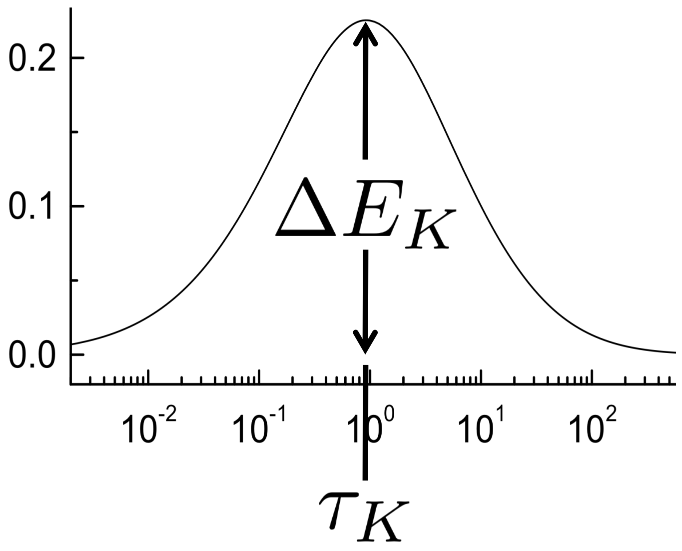


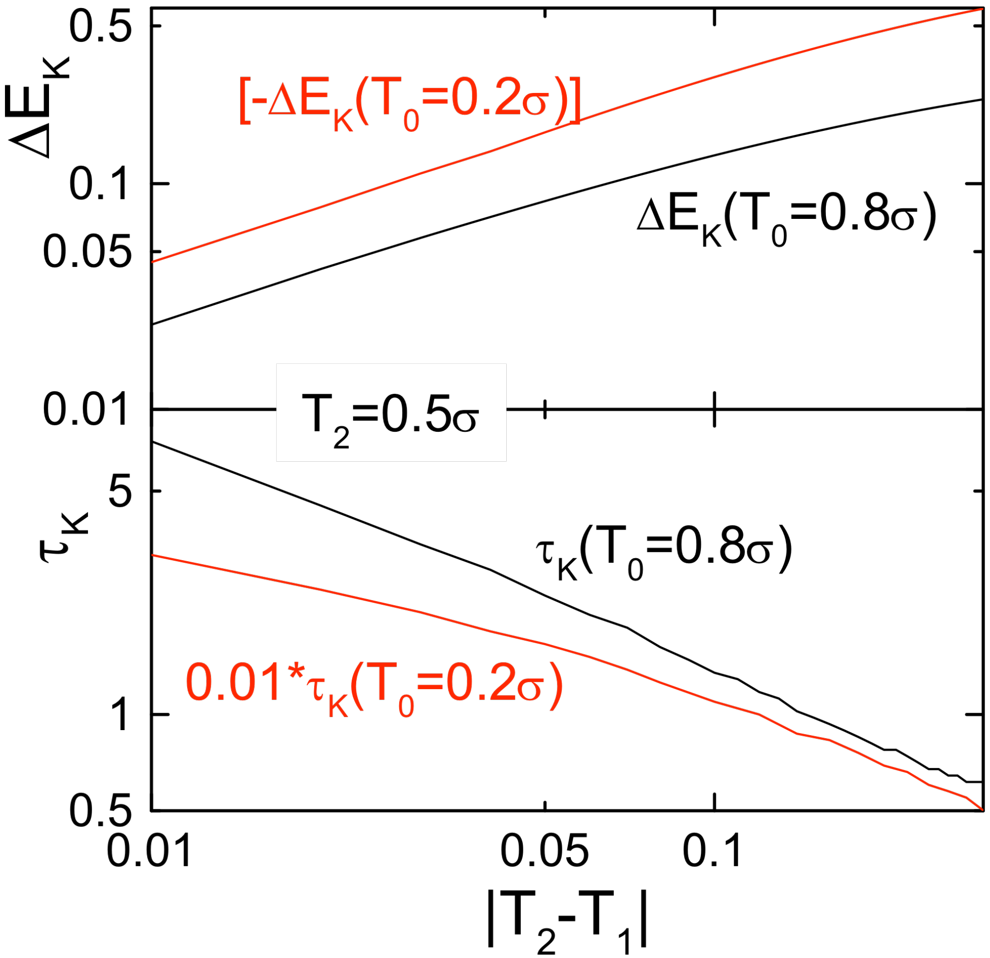






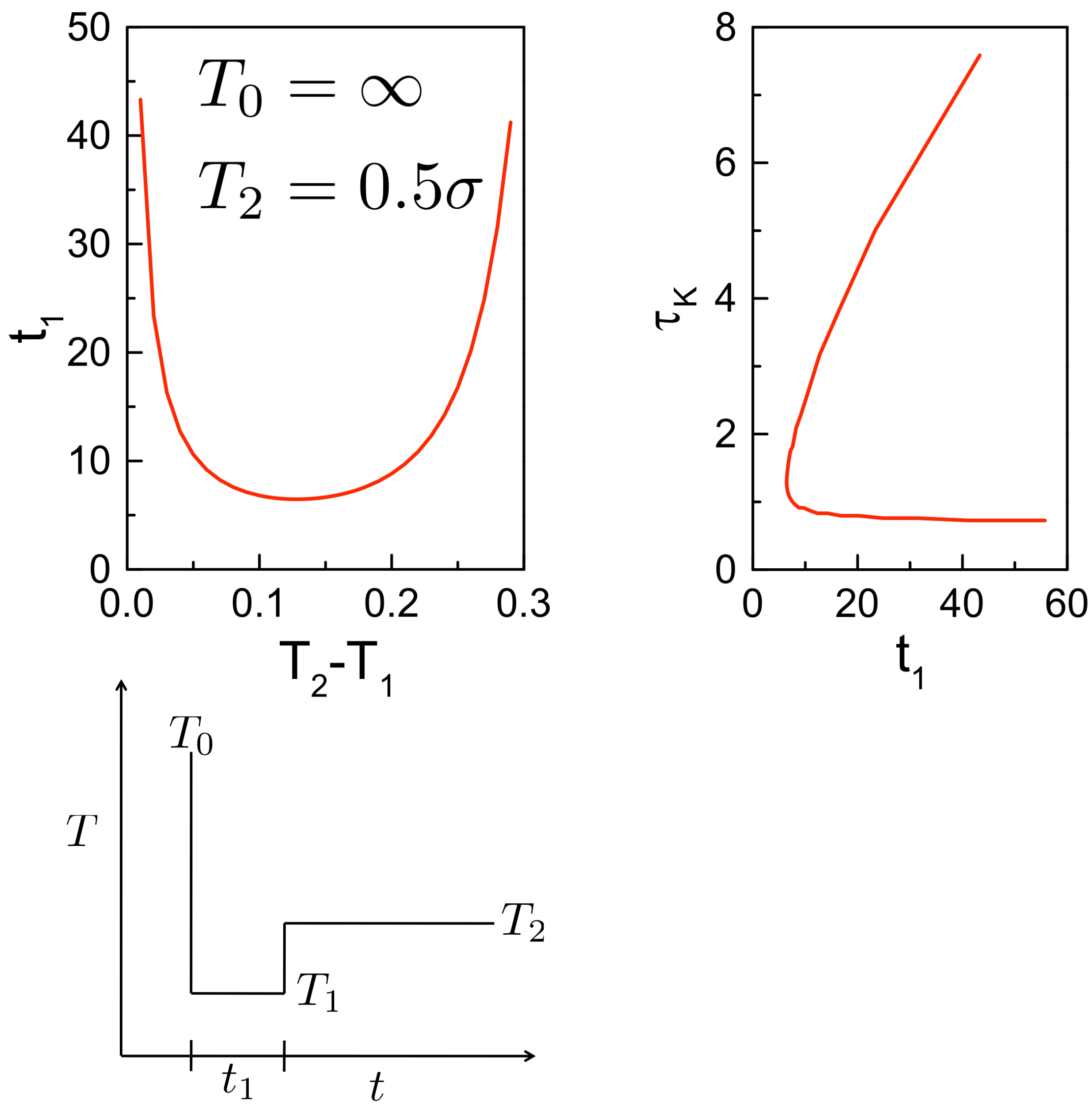
parameters:



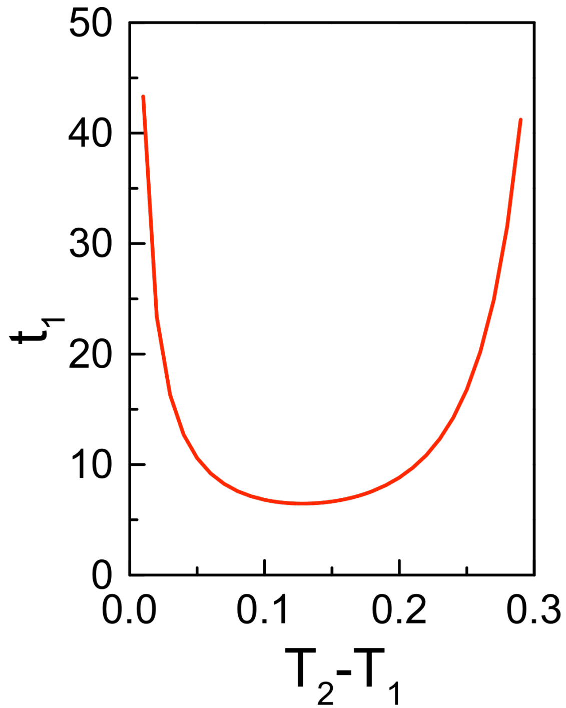


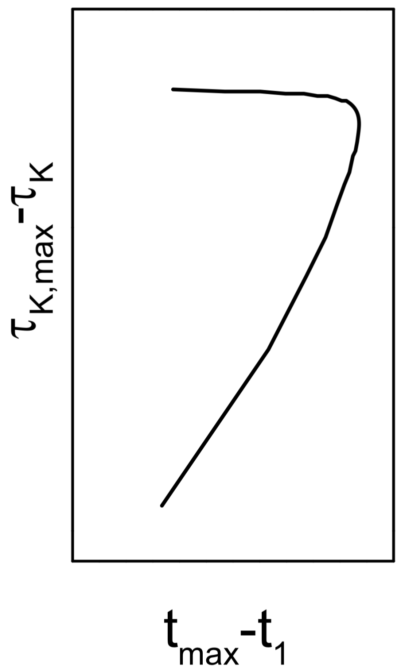
very similar behavior for quench and jump

unexpected behavior:



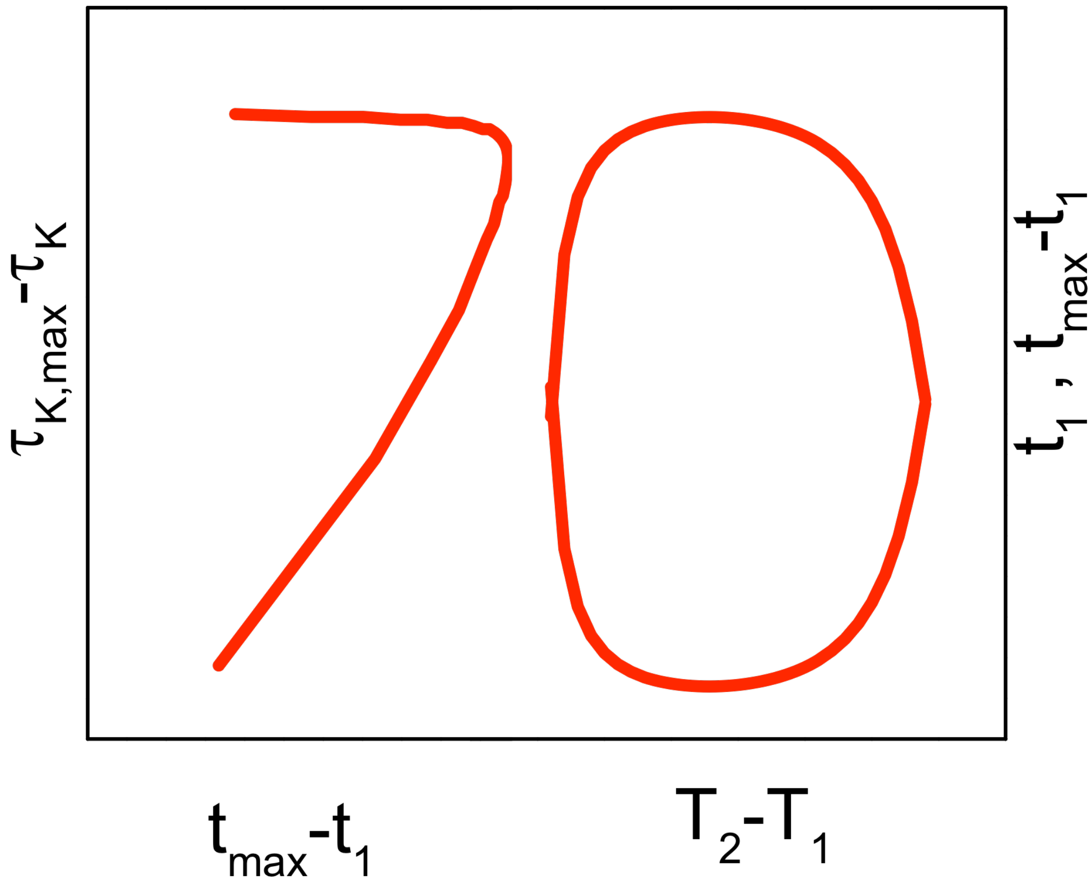
unexpected behavior:





unexpected behavior:

put as many information as possible in a single plot:



HAPPY BIRTHDAY, BOYE