

The stability of galaxies in an expanding Universe obtained by Newtonian dynamics

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Abstract

The dynamics of galaxies in an expanding Universe is often determined for gravitational and dark matter in an Einstein-de Sitter Universe, or alternatively by modifying the gravitational long-range attractions in the Newtonian dynamics. Here the time evolution of galaxies is determined by simulations of systems with pure gravitational forces by classical molecular dynamic simulations. A time reversible algorithm for formation and aging of gravitational systems by self-assembly of baryonic objects, recently derived (Toxvaerd 2022 *Eur. Phys. J. Plus* **137** 99), is extended to include the Hubble expansion of the space. The algorithm is stable for billions of time steps without any adjustments. The algorithm is used to simulate simple models of the Milky Way with the Hubble expansion of the Universe, and the galaxies are simulated for times which corresponds to more than 25 Gyr. The rotating galaxies lose bound objects from time to time, but they are still stable at the end of the simulations. The simulations indicate that the explanation for the dynamics of galaxies may be that the Universe is very young in cosmological times. Although the models of the Milky Way are rather stable at 13–14 Gyr, which corresponds to the cosmological time of the Universe, the Hubble expansion will sooner or later release the objects in the galaxies. But the simulations indicate that this will first happen in a far away future.

Keywords: stability of galaxies, simulation of galaxies, Age of the Universe

(Some figures may appear in colour only in the online journal)

1. Introduction

The constituents of the Universe are often found in clusters of objects, in planetary systems and in galaxies. The galaxies with stars were established shortly after the ‘Big Bang’ [1]. The two groupings of objects have a common feature, the objects rotate around their center of gravity.

But a rotating galaxy seems, however, to be unstable unless it contains a significantly larger mass (dark matter) than given by the known baryonic matter in the galaxy [2].

The Universe expands with a Hubble velocity [3], and the expansion of the Universe affects the dynamics of objects with gravitational forces. Simulations of galaxies with dark matter in an expanding Universe have been performed for many decades. In [4, 5] the authors solved the dynamics of baryonic objects by the particle–particle/particle–mess (PPPM) method [6], where each mass unit is treated as moving in the collective field of all others, and the Poisson equation for the PPPM grid is solved numerically. The dynamics are obtained for Zeldovich’s adiabatic expansion of the Einstein-de Sitter Universe [7]. Later, simulations with large scaled computer packages with PPPM [8, 9] are with many billions of mass units. The evolution of galaxies has also been obtained from hydrodynamical large scaled cosmological simulations [10–12], and with co-evolving dark matter gas and stellar objects [13, 14]. Cosmological simulations of galaxy formation is reviewed in [15].

An alternative theory for the dynamics of the galaxies, but without dark matter is the modified Newtonian dynamics (MOND), where Milgrom assumed that the very weak gravitational force experienced by a star in the outer regions of a galaxy varies inversely linearly with the distance, as opposed to the inverse square of the distance in Newton’s law of gravity [16]. Merging galaxies with MOND and with galaxies embedded in dark matter shows differences in the merging time-scales, but only very small differences in the final stellar distribution [17]. The MOND simulations techniques for modified gravity is given in [18]. The MOND theory is extensively tested and compared with the Lambda cold dark matter model and in favor of the MOND theory [19, 20].

Newtonian cosmology is derived in [21, 22]. Galactic dynamics with pure Newtonian gravity forces has been compared with N -body MOND simulations, and in favor of gravitational dynamics [23, 24], and Angus *et al* find similarly some shortcomings (mass-to-light ratios) of MOND at simulations of the Carina Dwarf Spheroidal Galaxy orbiting the Milky Way [25]. The Newtonian dynamics simulations of galaxy systems described below complement the existing analyzes of the stability of galaxies. The N -body simulations are for models of celestial objects in the Milky Way, and the objects interact with pure gravitational forces and with and without the Hubble expansion of the space.

The dynamics of models for galaxies are based on an algorithm for the classical dynamics of systems of celestial objects [26]. Here (see [appendix](#)) the algorithm is extended to include the Hubble expansion of the space. The discrete algorithm is time reversible and stable for billions of time steps without any adjustments. The algorithm is used to simulate simple models of the Milky Way with and without the Hubble expansion of the Universe. The different models of galaxies are simulated for times which correspond to more than twice the age of the Universe and both the galaxies without- as well as galaxies with the Hubble expansion of the space are stable. The simulations indicate that an alternative explanation to the dynamics of galaxies with dark matter is, that the Universe is very young in cosmological time units given by one rotation of the galaxy. A galaxy as old as our Universe lose bound objects, but although the Hubble expansion sooner or later will release the stars in the Milky Way, the simulations show that it will first happen in the distant future.

2. The expanding Universe

A galaxy, I , far away in the Universe moves away from the Earth at a speed proportional to its ‘proper distance’, $Hr_{kl}(t)$, to the Earth, k , measured at the ‘cosmological time’ t , and this

behavior is explained by an expanding Universe. The Hubble constant H is the expansion coefficient in Hubble's law [3, 27]

$$\mathbf{v}^H(\mathbf{r}_{kl}(t)) = H\mathbf{r}_{kl}(t) \quad (1)$$

for the velocity of the increase of the distance $\mathbf{r}_{kl}(t)$ from the Earth to a galaxy. The Hubble expansion can be obtained by an intensive expansion of the space independently of the baryonic matter in the Universe

$$\mathbf{v}^H(\mathbf{r}(t)) = H\mathbf{r}(t), \quad (2)$$

by which the distance between pairs of positions $\mathbf{r}_k(t), \mathbf{r}_l(t)$, or $\mathbf{r}_k(t), \mathbf{r}_k(t + \delta t)$ increases with the Hubble velocity. Equation (2) fulfills the *Cosmological principle* and the *Copernican principle* for expansion of the Universe [28]. The Cosmological principle, which was first formulated by Newton [29] demands, that no place in the Universe is preferred, and the Copernican principle demands, that no direction in the Universe is preferred. Equation (2) will accelerate the expansion of the Universe, and observations of galaxies indicates, that the distances between the galaxies accelerate (dark energy) [30, 31].

The Hubble constant H is quoted in $\text{km s}^{-1} \text{Mpc}^{-1}$, for the velocity in km s^{-1} of a galaxy l megaparsec ($3.09 \times 10^{19} \text{ km}$) away. Its value is [32]

$$H = 72.1 \pm 2.0 \text{ km s}^{-1} \text{Mpc}^{-1}. \quad (3)$$

For the Solar System with distances in au ($1 \text{ au} \approx \text{Earth's distance to the Sun} = 149\,597\,870.700 \text{ km}$ (definition)) the Hubble constant is $33.93 \times 10^{-11} \text{ km s}^{-1} \text{au}^{-1}$, and the Earth's mean velocity in the Solar System is $\approx 30 \text{ km s}^{-1}$, so the Hubble velocity is about $\approx 10^{-11}$ of the Earth's velocity. This small Hubble velocity has no effect on the orbits of the planets and the stability of a planetary system (see later).

The Milky Way is a barred spiral galaxy. The extension of the barred disk is 2.5–3 kpc [33], and the extension of the halos is $\approx 100\,000 \text{ pc}$, but some recent investigations indicate that this might be substantial larger, $\approx 300 \text{ kpc}$ or even more [34, 35]. The Milky Way rotates with a rotational velocity $220 \pm 10 \text{ km s}^{-1}$ for stars at a distance of $8.0 \pm 0.310 \text{ kpc}$ from its center [36], but the rotations at different distances from the center of the galaxy deviate from calculations of the rotation caused by the gravitational forces between its constituents, and this deviation is explained by the existence of unknown dark matter [2, 37, 38]. The Milky Way contains very old stars from the period just after the 'Big Bang' [39], so the galaxy has presumably exist for more than 13 billion years. The rotational time (galactic year) of the Milky Way is estimated to be $\approx 220\text{--}240$ million years, and this implies that an old star in the Milky Way with this rotational velocity must have performed of the order ≈ 60 cycles.

The Hubble velocity between pairs of stars in the Milky Way at distances of e.g. $5 \times 10^{-2} \text{ Mpc}$ is 3.5 km s^{-1} , and it is not negligible compared with the rotational velocity of the galaxy. The expansion of the Universe is with a Hubble velocity, which is of the order $\approx 1\%$ or more of the rotational velocity of the Milky Way, and the Hubble expansion will affect the rotation and the stability of the galaxy.

3. Simulation of galaxies in an expanding Universe

A galaxy and the Milky Way contains hundred of billion of stars, and a substantial amount of baryonic gas [40, 41] and it is not possible directly to obtain the Newtonian dynamics of a galaxy with this number of objects. Instead, we have simulate models of small 'galaxies' of

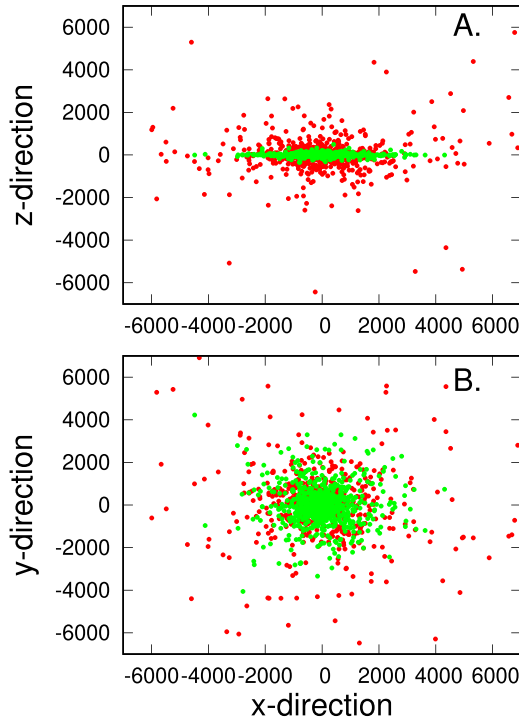


Figure 1. Positions of the objects. (A) Site view of the system; (B) top view. The green dots are the positions at the start and the red dots are the positions at $t = 2.5 \times 10^4$ (10^7 time steps).

hundred of objects in orbits around their center of gravity, and in an expanding space with various strength of the expansion. A recent article describes how to create a collection of objects with rotation about their center of gravity by molecular dynamics (MD) simulations, obtained by an extension of Newton's discrete algorithm [26]. The algorithm is further developed to also include the Hubble expansion of space, and the algorithm with the extensions are given in the [appendix](#).

The collection of baryonic objects with Newtonian dynamics will, for many starting positions and velocities of the objects spontaneously form a system with many of the objects in rotations about a heavy mass in its center. If these MD systems shall simulate the dynamics of a galaxy in the expanding Universe, then one must relate *distances*, *times*, and *Hubble expansion* in the MD systems with the corresponding distances, times and Hubble expansion in a galaxy.

3.1. Determination of distances, times and H for the Hubble expansion in the MD systems

A system *without a Hubble expansion* of the space is obtained from ensembles of $N = 1000$ objects with a disk-shaped distribution of their start positions and with \approx zero velocities in the z -direction. The distribution of the objects at the start and after the creation of a system with ≈ 300 of the objects in circulations around a heavy object at the center of mass is shown in figure 1.

The system with positions shown in figure 1 is simulated for a long period of time. The system is evolving and with mergers of objects, and after $t = 2.5 \times 10^6$ it contains $N = 557$

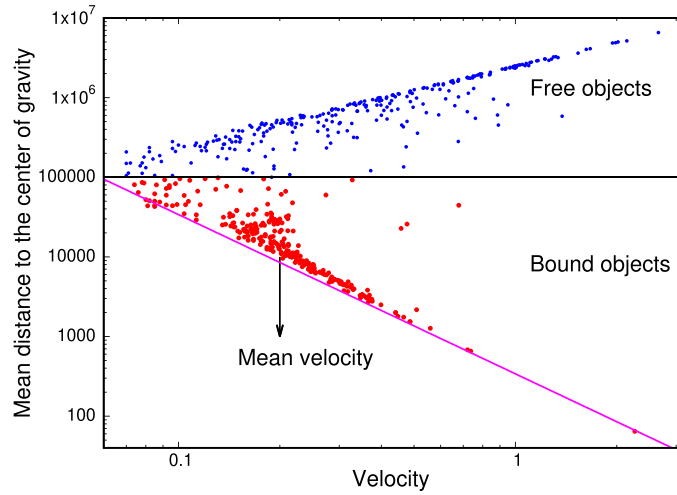


Figure 2. log mean distances to the mass center as a function of the log mean velocities of the objects. The bound objects (red spheres) are located on the lower branch of the distribution and the upper branch shows (blue spheres) the mean locations of the free objects. Magenta line: $\log(y) = \log(Gm_c/x^2)$.

objects, and with 270 of them in circulations around the heavy mass in the center. The mean distances, $\bar{r}_{i,c}$, in the succeeding time interval $\Delta t \in [2.5 \times 10^6, 3.0 \times 10^6]$, as a function of the mean velocity, $\bar{v}_{i,c}$, relative to the center c is shown in figure 2. A bound object will have a relative small mean distance to the center of mass in contrast to a free object, which has a large and monotonic increasing distance to the center. The lower branch of mean distances in figure 2 are for the collection of bound objects, and the upper branch are the mean distances of the free objects moving away from the center with constant velocities. An object, i , in an elliptic orbit around a heavy gravitational center with the mass m_c will have a mean velocity $\bar{v}_{i,c} = \sqrt{Gm_c/a_i}$, where a_i is the semimajor axis in its ellipse. The magenta line is the line $\log(y) = \log(Gm_c/x^2)$ with the mass $m_c = 340$ of the center of the system (for units in the MD systems see the [appendix](#)). The systematic deviation of the mean distances of the bound objects from the straight line for increasing distances to the center is partly caused by, that the eccentricities of the orbits of the bound objects increase with increasing distance to the center of mass, but the distribution of objects for greater distances $2000 < \bar{r}_{i,c}$ shows, that the objects are not lined up in a row of objects in elliptical orbits as in a planetary system.

The mean distances, rotation times and velocities for the bound objects, shown in figure 2 are used to relate the MD system with the Milky Way.

The Milky Way has an extension of ≈ 100000 pc, but the extension of some of the halos may be more. The lower branch in figure 2 with the bound objects is within a MD distance 100000 MD length units or less from the center. So a relation between the distances in the Milky Way in pc and the distances in the MD system is $1 \text{ ps} = 1 \text{ MD length unit}$.

The galactic year of the Milky Way is about 220 million years [36], and the Milky Way contains a very old star [39], which has been created shortly after the ‘Big Bang’. A star with a rotation time of 220 million years, must have performed only ≈ 60 cycles in the Milky Way after it is created. The distance R_0 of our planet system to the center of the Milky way is $R_0 = 8.34 \pm 0.16 \text{ kpc}$ [42]. These data are used to relate the cosmological time with the time in the MD systems. The rotation time of objects in the MD system at ≈ 8000 MD length units

is $t_{\text{orbit}} \approx 1.3 \times 10^5$, so these objects have performed ≈ 60 rotations at the MD time 8.0×10^6 . The figure 2 is for 0.3125 of the cosmological time of the Universe, or ≈ 4.2 Gyr, and the age of the Universe corresponds to $\approx 8.0 \times 10^6$ in the MD systems.

The mean velocity \bar{v} of the bound objects in the MD system in figure 2 is $\bar{v} \approx 0.15$. The value of the Hubble expansion constant in the MD system is obtained from the ratio H/\bar{v} between the Hubble constant per unit length and the rotational velocity of the galaxy as

$$(H/\bar{v})_{\text{MD}} = (H/\bar{v})_{\text{Milky Way}}, \quad (4)$$

which determines the value of H in the MD system,

$$\begin{aligned} H &= 0.15 \times 72 \times 10^{-6} \text{ km s}^{-1} \text{ pc}^{-1} / 220 \text{ km s}^{-1} \\ &\approx 5 \times 10^{-8} \text{ per MD length unit.} \end{aligned} \quad (5)$$

Let the estimates of the extent of the Milky Way, the extent of the MD model of a galaxy, the rotation time of the Milky Way and the mean velocity in the MD system all be with an uncertainty with a factor of 2, then the total uncertainty of the estimate of H in the MD systems is $\approx 2^4 \approx 10$. The value of the Hubble constant in the MD units for the model of the Milky Way in the expanding Universe is

$$H = 5 \times 10^{-8 \pm 1}. \quad (6)$$

3.2. The formation and stability of galaxies obtained by classical dynamics

The MD system without an expansion of the space, with positions shown in figure 1 at $t=0$ (green) and $t=2.5 \times 10^4$ (red), and figure 2 at $t=2.5 \times 10^6$ is formed spontaneously. This system is used to investigate the formation and stability of MD systems to various values of H in the Hubble expansion. Systems with $H=0$, 5×10^{-8} , 5×10^{-7} and 5×10^{-6} , respectively, are simulated and their behavior is shown below.

In figure 3 the distributions of objects at $t=2.5 \times 10^6$ is shown for various strength of the Hubble constant. The four systems are started with the disk formed distribution of 1000 objects with equal masses, shown with green dots in figure 1. All four systems spontaneously formed gravitational systems. The systems contain many bound objects with different masses and in circulation around their center of gravity, but the system with $H=5 \times 10^{-6}$ was unstable and with no bound objects already after $t=1.0 \times 10^6$. The radius of the disk in the Milky way is ≈ 15 kpc [33] corresponding to a distance 15 000 MD length units in the MD systems. At $t=2.5 \times 10^6$, the number of objects within this distance to the center of the galaxy are 144, 152, 19 and 0, respectively for the four different values of the expansion. The system with $H=5 \times 10^{-7}$ and only 19 objects with $\bar{r}_{i,c} < 15000$ is, however, also unstable and without any objects with distances less than 15 000, but first after $t=6.5 \times 10^6$ (figure 4).

Figure 5 shows the time evolution of the number of objects within a mean distance $\bar{r}_{i,c} < 15000$ from the center c . Almost all of the objects are within this distance at the start of the evolution, but as the self-assembly and aging of the Newtonian systems evolves, the number declines slowly for $H \leq 5 \times 10^{-8}$, whereas it rapidly goes to zero for faster expansions. The arrow to the left marks the time 2.5×10^6 with the distribution of objects shown in figure 3, and the arrow in the middle is at the time 8.0×10^6 , which should correspond to the age of our Universe. The systems with $H \leq 5 \times 10^{-8}$ are rather stable at this time, but with a weak declining number of objects within the distance $\bar{r}_{i,c} < 15000$. The systems are started from the same start distribution, shown with green spheres in figure 1, and the self-assembly with merging of objects results in slightly different masses of the center of the galaxies and small differences

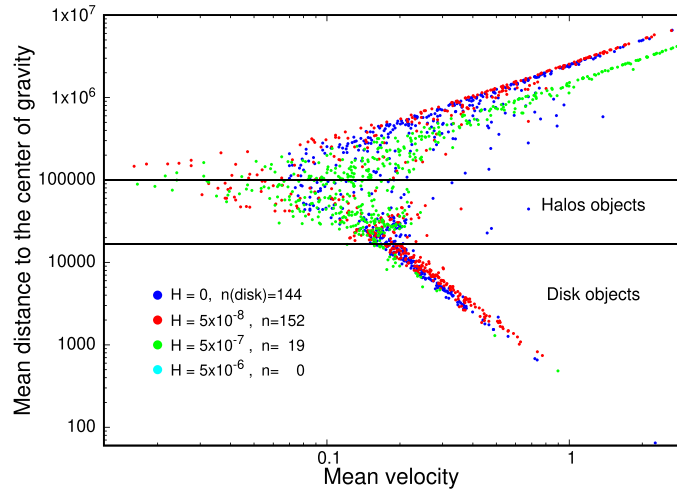


Figure 3. The log mean distances $\log(\bar{r}_{i,c})$ of the objects as a function of their log mean velocities $\log(\bar{v}_{i,c})$ with respect to the center c for different values of the Hubble constant H . The mean positions are obtained for the time interval $\Delta t \in [2.5 \times 10^6, 3.0 \times 10^6]$.

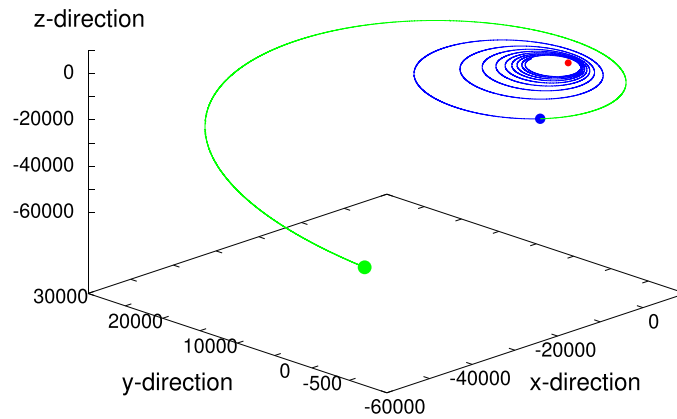


Figure 4. The release of the last bound object I from the system with $H = 5 \times 10^{-7}$. Blue: the orbit for $\bar{r}_{l,c}(t) < 15000$ and green: for $\bar{r}_{l,c}(t) > 15000$. The center of gravity with red. The blue sphere is the position at $t = 6.5 \times 10^6$, and the green sphere is the position at $t = 7.75 \times 10^6$.

between the numbers for $H = 0$ and $H = 5 \times 10^{-8}$. The left inset in figure 5 shows the L_z components of the angular momenta \mathbf{L} for the galaxies with the different strength of the expansion. The angular momentum of a system without a Hubble expansion is conserved for Newton’s discrete dynamics (blue curve in the inset), but the Hubble expansion increases the angular momenta, and this will sooner or later destroy the galaxies (figure 4). The number of bound object with mean distance $\bar{r}_{i,c}(t) < 15000$ to the center of the galaxy with $H = 5 \times 10^{-8}$ is compared with the corresponding total number of all bound objects (objects with $\bar{r}_{i,c}(t) < 10^5$) in the galaxy in the right inset of figure 5. The two sets of numbers decreases with time, but

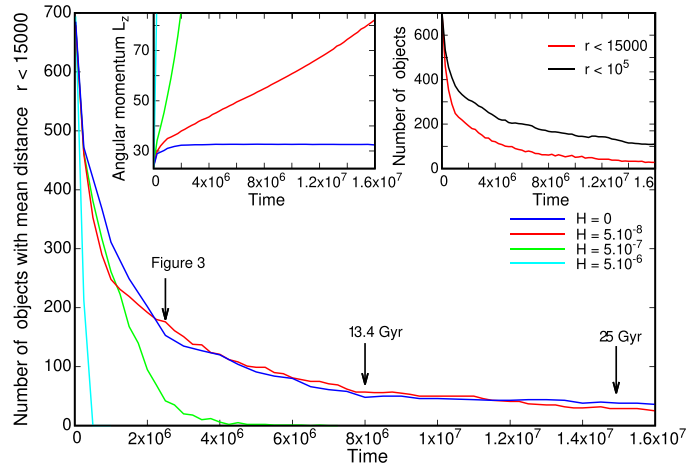


Figure 5. Number of objects i with mean distances $\bar{r}_{i,c}(t) < 15000$ to the center of the galaxy c as a function of time. The systems are started from the ‘gas’ distribution shown with green dots in figure 1. The colors (same as in figure 3) are for different strength of the Hubble expansion. The inset to the left shows the time evolution of the L_z -components of the angular momenta of the galaxies with different strength of the Hubble expansion. In the right inset the number of bound objects with $\bar{r}_{i,c}(t) < 15000$ for $H = 5 \times 10^{-8}$ are compared with the total number of bound objects $\bar{r}_{i,c}(t) < 10^5$ in the galaxy.

very slowly and the destruction of the system with first happen in a far away future for the galaxy with $H = 5 \times 10^{-8}$.

A series of control simulations have been performed in order to ensure, that the results shown in figure 5 are representative for the formation and aging of gravitational systems with Hubble expansion. In one set of test simulations the three different expansions was first included in the dynamics at $t = 2.5 \times 10^6$ in the system without expansion. But with the same result that the system with $H = 5 \times 10^{-8}$ was rather stable, but the two systems with stronger Hubble expansions were unstable. In another set of simulations the Hubble expansion was added to a Newtonian system with another distribution of the objects at the start of the formation of the galaxy, but with the same result: all the systems are simulated for times much longer, that corresponds to ours cosmological time ≈ 13.4 Gyr, and the systems are still rather stable for an expansion with $H \leq 5 \times 10^{-8}$, which should correspond to the Hubble expansion of the Universe.

Figure 6 shows the log mean distances $\log(\bar{r}_{i,c})$ of the objects as a function of their log mean velocities $\log(\bar{v}_{i,c})$ with respect to the center c of the system with $H = 5 \times 10^{-8}$, and at different times. With red is for $t = 2.5 \times 10^6$, also shown in figure 3, and with green is for $t = 8 \times 10^6$ (≈ 13.4 Gyr). The numbers N_{disk} are for objects with $\bar{r}_{i,c} < 15000$ and the numbers N_{halos} are for objects with $15000 < \bar{r}_{i,c} < 100000$. $\langle \bar{r}_{i,c} \rangle$ is the mean distances for the $N_{\text{disk}} + N_{\text{halos}}$ objects. The blue line, $\log(r) = \log(Gm_c/v^2)$ is the line for mean distances of objects in ellipses with the same eccentricities. The distributions of the objects deviates from this line for a ‘Kepler-like’ order with the objects in unperturbed elliptical orbits around the center of mass. In particular, the objects with large distances to the center of gravity deviate from the straight line by having in general a higher velocity than given by the blue line, and the aging of the system does not remove this tendency. The system at $t = 8 \times 10^6$ should correspond to the cosmological time 13.4 Gyr of the Universe, and the systems are still aging with losses of

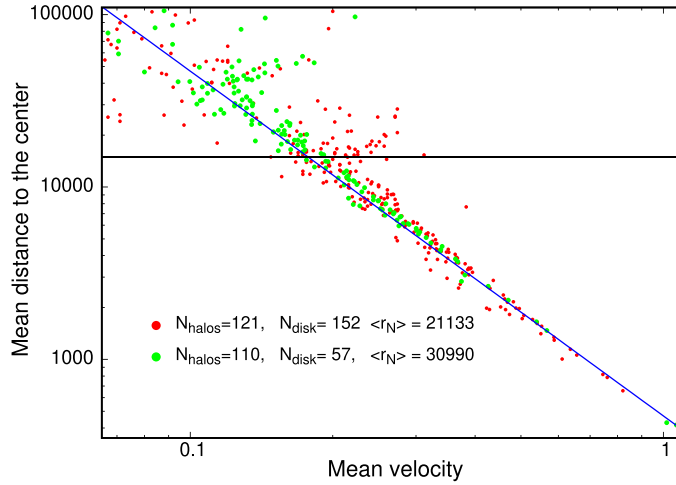


Figure 6. $\log(\bar{r}_{i,c})$ as a function of $\log(\bar{v}_{i,c})$ and at different times. With red is for $t = 2.5 \times 10^6$, and with green is for $t = 8 \times 10^6$ (≈ 13.4 Gyr). N_{disk} are for $\bar{r}_{i,c} < 15000$ and N_{halos} are for $15000 < \bar{r}_{i,c} < 100000$. $\langle r_N \rangle$ is the mean positions for the $N_{\text{disk}} + N_{\text{halos}}$ objects. The blue line, $r = m_c/v^2$, is the line for mean positions of objects in ellipses, and with the same eccentricities.

bound objects and with a monotonic increasing rotational velocities, as shown in the inset of figure 5 for $L_z(t)$.

4. Discussion

The dynamics of galaxies in an expanding Universe are often determined for gravitational and dark matter in an Einstein-de Sitter Universe, or alternatively by the MOND [16]. But here the dynamics of galaxies is obtained for systems with pure gravitational matter by classical MD simulations. The dynamics of the systems is not explained by dark matter, nor by a modification of the gravitational attractions at long distances, but by that the Universe is a young Universe measured in cosmological times. The Milky Way is believed to be more than 13 Gyr, but it corresponds to only ≈ 60 rotations or galactic years, and it is nothing compared to the age of our Solar System with 4.56 Gyr, i.e. number of rotations of the Earth. The stability of the Solar System is still to debate [43, 44], and in the light of these facts the Universe must be characterized as a very young Universe. The dynamics and aging of the MD systems with gravitational matter shows, however, that the objects in the small MD systems are not in a stable steady state after only sixty rotations, so how can hundred of billion of stars and a substantial amount of baryonic gas then be it?

According to the present investigation, the explanation for the dynamics of the Universe could be, that the galaxies are formed spontaneously at the relatively high concentration of baryonic matter shortly after the Big Bang. The Universe expands and the galaxies are aging, but the galaxies are not in a steady state at a time which corresponds to 13.4 Gyr (figures 5 and 6). The systems with- and without a Hubble expansion still occasionally lose bound objects at 13.4 Gyr, and the mean distance of the bound objects increase also over time. The bound objects far away from the center of gravity are not distributed as in a stable Newtonian system. They have in general a higher velocity, than if they were ordered in simple Kepler-like

orbits (figure 6). The hypothesis can perhaps be tested by statistical analyses of the density and velocity distribution (distribution of light intensity) in galaxies as functions of the age of the galaxies.

The Hubble expansion increases the angular momentum of a galaxy and this fact implies, that the galaxy sooner or later will be destroyed, and that all the objects will be free in an open Universe. The end of the system with $H = 5 \times 10^{-7}$ is shown in figure 4, with the release of the last bound object in the system. But this system has a strength of the Hubble expansion, which probably is ten times stronger than in the Universe. The simulations indicate, however, that for the systems with a Hubble expansion similar to the Hubble expansion of the Universe, this destruction might first happen in a far away future. These systems are simulated for what corresponds to more than 25 Gyr, and they still contain many objects with $\bar{r} < 15\,000$ (figure 5).

The algorithm is well suited for analyzes of formation and aging of gravitational systems in an expanding Universe. The central difference algorithm, equation (A.11) + (A.3), is time reversible, and the underlying Newtonian discrete algorithm, which is used in almost all MD simulations in physics and chemistry is exact in the sense, that it has the same invariances as Newton's analytic dynamics [26, 45]. The algorithm is stable, the present simulations are with more than 6×10^9 discrete time steps for each systems, and each simulation has taken four to five months, but without any adjustments.

Data availability statement

Data will be available on request.

The data that support the findings of this study are available upon reasonable request from the authors.

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Appendix. A time reversible algorithm for discrete classical dynamics in an expanding space

Newton's discrete central difference algorithm

In Newton's classical discrete dynamics [29, 46] a new position $\mathbf{r}_k(t + \delta t)$ at time $t + \delta t$ of an object k with the mass m_k is determined by the force $\mathbf{f}_k(t)$ acting on the object at the discrete positions $\mathbf{r}_k(t)$ at time t , and the position $\mathbf{r}_k(t - \delta t)$ at $t - \delta t$ as

$$m_k \frac{\mathbf{r}_k(t + \delta t) - \mathbf{r}_k(t)}{\delta t} = m_k \frac{\mathbf{r}_k(t) - \mathbf{r}_k(t - \delta t)}{\delta t} + \delta t \mathbf{f}_k(t), \quad (\text{A.1})$$

where the momenta $\mathbf{p}_k(t + \delta t/2) = m_k(\mathbf{r}_k(t + \delta t) - \mathbf{r}_k(t))/\delta t$ and $\mathbf{p}_k(t - \delta t/2) = m_k(\mathbf{r}_k(t) - \mathbf{r}_k(t - \delta t))/\delta t$ are constant in the time intervals in between the discrete positions. Newton postulated equation (A.1) and obtained his second law, and the analytic dynamics in the limit $\lim_{\delta t \rightarrow 0}$.

The algorithm, equation (A.1), is usual presented as the 'Leap frog' algorithm for the velocities

$$\mathbf{v}_k(t + \delta t/2) = \mathbf{v}_k(t - \delta t/2) + \delta t/m_k \mathbf{f}_k(t). \quad (\text{A.2})$$

The positions are determined from the discrete values of the momenta/velocities as

$$\mathbf{r}_k(t + \delta t) = \mathbf{r}_k(t) + \delta t \mathbf{v}_k(t + \delta t/2). \quad (\text{A.3})$$

Newton's discrete algorithm, equation (A.1), has been re-derived many times and with different names, and almost all molecular dynamics (MD) simulations in physics and chemistry as well as many simulations in astrophysics are performed with his discrete algorithm. The rearrangement of equation (A.1) gives e.g. the 'Verlet' algorithm [47].

The classical discrete dynamics between N spherically symmetrical objects with masses $m^N = m_1, m_2, \dots, m_N$ and positions $\mathbf{r}^N(t) = \mathbf{r}_1(t), \mathbf{r}_2(t), \dots, \mathbf{r}_N(t)$ is obtained by equations (A.2) and (A.3). The momentum and angular momentum for a conservative system of the N objects is conserved due to Newton's third law, and the energy is also conserved [45, 48, 49], so Newton's time reversible and symplectic discrete dynamics has the same invariances as his analytic dynamics. Newton's discrete algorithm make it possible to simulate systems of interacting objects for billions of time steps without *any* adjustments for the infinite range of the gravitational forces or corrections for symplecticity, time reversibility, conservation of momentum, angular momentum and energy.

Newton's discrete central difference algorithm with merging of gravitational objects

The discrete central difference algorithm for merging of objects is derived in [26]. Let all the spherically symmetrical objects have the same (reduced) number density $\rho = (\pi/6)^{-1}$ by which the diameter σ_i of the spherical object i is

$$\sigma_i = m_i^{1/3} \quad (\text{A.4})$$

and the collision diameter

$$\sigma_{ij} = \frac{\sigma_i + \sigma_j}{2}. \quad (\text{A.5})$$

If the distance $r_{ij}(t)$ at time t between two objects is less than σ_{ij} the two objects merge to one spherical symmetrical object with mass

$$m_\alpha = m_i + m_j, \quad (\text{A.6})$$

and diameter

$$\sigma_\alpha = (m_\alpha)^{1/3}, \quad (\text{A.7})$$

and with the new object α at the position

$$\mathbf{r}_\alpha(t) = \frac{m_i}{m_\alpha} \mathbf{r}_i(t) + \frac{m_j}{m_\alpha} \mathbf{r}_j(t), \quad (\text{A.8})$$

at the center of mass of the two objects before the fusion. (The object α at the center of mass of the two merged objects i and j might occasionally be near another object k by which more objects merge, but after the same laws.)

The momenta of the objects in the discrete dynamics just before the fusion are $\mathbf{p}^N(t - \delta t/2)$ and the total momentum of the system is conserved at the fusion if

$$\mathbf{v}_\alpha(t - \delta t/2) = \frac{m_i}{m_\alpha} \mathbf{v}_i(t - \delta t/2) + \frac{m_j}{m_\alpha} \mathbf{v}_j(t - \delta t/2), \quad (\text{A.9})$$

which determines the velocity $\mathbf{v}_\alpha(t - \delta t/2)$ of the merged object.

The algorithm for the system of baryonic objects consists of the equations (A.2) and (A.3) for time steps without merging of objects, and the fusion of objects is given by the

equations (A.5)–(A.9). The discrete algorithm with merging of objects has the same invariances as Newton’s exact analytic and his discrete dynamics [26].

The algorithm for discrete classical mechanics in the expanding Universe

Newton’s discrete dynamics changes the position of an object k . If the space expands monotonously over time with the Hubble velocity \mathbf{v}^H , then the expansion also changes the distance between two positions. The new position $\mathbf{r}_k(t + \delta t)$ is the sum of the change due to the gravitational force on k and the contribution from the Hubble expansion. The mean location of an object changes from $\mathbf{r}_k(t - \delta t/2) = (\mathbf{r}_k(t - \delta t) + \mathbf{r}_k(t))/2$ at $t \in [t - \delta t, t]$ to $\mathbf{r}_k(t + \delta t/2) = (\mathbf{r}_k(t) + \mathbf{r}_k(t + \delta t))/2$ at $t \in [t, t + \delta t]$. The Hubble expansion changes the distance between the two positions by the Hubble velocity

$$\begin{aligned} \mathbf{v}^H &= H\delta\mathbf{r}_k = H(\mathbf{r}_k(t + \delta t/2) - \mathbf{r}_k(t - \delta t/2)) \\ &= \delta t H \frac{\mathbf{r}_k(t + \delta t) - \mathbf{r}_k(t)}{2\delta t} + \delta t H \frac{\mathbf{r}_k(t) - \mathbf{r}_k(t - \delta t)}{2\delta t} \\ &= \frac{\delta t H}{2} \mathbf{v}_k(t + \delta t/2) + \frac{\delta t H}{2} \mathbf{v}_k(t - \delta t/2). \end{aligned} \quad (\text{A.10})$$

By including the Hubble velocity, equation (A.10), in the Newton’s algorithm, equation (A.2), and after a re-arrangement, one obtains the algorithm for the classical mechanics with a Hubble expansion of the space included in the Newtonian dynamics

$$\mathbf{v}_k(t + \delta t/2) = \frac{(1 + \delta t H/2)\mathbf{v}_k(t - \delta t/2) + \delta t/m_k \mathbf{f}_k(t)}{1 - \delta t H/2}. \quad (\text{A.11})$$

The discrete classical dynamics with Hubble expansion, equations (A.11) and (A.3), is still time reversible, but it increases the velocities, the momenta and the angular momenta.

The small galaxies in the articles are obtained for thousand objects, which at the start of the simulation, at $t=0$ are separated with a mean distance $\langle r_{ij}(0) \rangle \approx 1000$ and with a Maxwell–Boltzmann distributed velocities with mean velocity $\langle |\mathbf{v}_i(0)| \rangle \approx 1$. The gravitational strengths in the article are in units of the gravitational constant $G=1$ and the mass unit $m = m_i(0) = 1$ and diameters of the objects $\sigma_i(0) = 1$. For the set-up of the systems see also [26]. The systems are followed more than 6.4×10^9 MD time steps, i.e. $t = 1.6 \times 10^7$ time-units, which corresponds to 25 Gyr of our Universe.

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